Cosmological constraints on neutrinos beyond the standard model

Lachlan Lancaster

ltlancas@andrew.cmu.edu

McWilliams Center for Cosmology, Department of Physics, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA 15213

ABSTRACT

Using several constraining data sets, we consider the effects of scalar-mediated, non-standard model neutrino interactions on cosmological observables. Specifically, we consider the limit of a very massive mediating scalar particle ϕ, which mediates a Fermi-like four-fermion interaction between neutrinos. This interaction is characterized by an effective coupling constant G_{eff}, which is analogous to the Fermi constant G_F of the electroweak interaction. Using only CMB data, we find a multimodal posterior distribution which has modes which both favor and disfavor a strong neutrino interaction of the form we study here. The non-interacting mode places limits on the value of $\log_{10}\left(G_{eff}\text{Mev}^2\right) < -3.47$ at (95% C.L.). More interestingly, we find an interacting mode with a value of $\log_{10}\left(G_{eff}\text{Mev}^2\right) = -1.72 \pm 0.099$ (68% C.L.), suggesting an interaction even stronger than previous results is favored by recent Planck Data. However, using recent SDSS DR11 data, we are able to exclude this mode almost entirely. Further, we use a Fisher Matrix Analysis to predict the ability of future experiments, including the CMB S4 studies, to put limits on this type of interaction (Wu et al. 2014) . Finally, we discuss possible generalizations to the model presented here as well as the general viability and implications of this type of interaction.

Subject headings: methods: data analysis — techniques: Bayesian Inference

1. Introduction

1.1. Background in the Cosmos

The Cosmic Microwave Background (CMB) radiation is light which has traveled from near the time of the big bang to reach us today, and makes up most of the energy in the form of light that exists in the universe. In the early universe there were no atoms or larger strucutres, the whole universe consisted of an ionized soup of particles that was very nearly evenly distributed in space.

That is to say that this early fluid was *nearly* homogeneous (it looked the same wherever you were in space) and isotropic (it looked the same in every direction in space). However, small inhomogeneities and anisotropies created in the very early universe caused this fluid of particles, such as protons, neutrons, electrons, photons, and other fundamental particles, to evolve. This evolution consisted mainly of growth of the over-dense regions, due to gravity, and diffusion of these regions, due to pressure from photons on the electrically charged particle soup. This balancing act between gravity and photon pressure created acoustic waves in the density of this primordial soup.

At this time the universe was so dense that the light that was moving around could not move very far at all before scattering off of a nearby electron. We descrube this by saying that the universe was 'opaque,' as the light could not move freely. This light actually made up the majority of the energy density of the universe at the time, (most likely due to the extremely early time anihilation of matter with anti-matter), and thus had a large effect on the evolution of this energy density. Since this light could not travel very far without scattering off of something we know that this light had to 'know' a lot about the matter that it was interacting with. We can then say that this light was in thermal equilibrium with its surroundings. This is good because we then know from Max Planck what the spectrum of this light should look like: a blackboady spectrum. This spectrum is determined by a single parameter, the temperature of the 'body.'

As the universe expanded it gradually became less and less dense to the point where light stopped meeting electrons in its travels and eventually just kept going. This is the time in the universe that is referred to as the epoch of last-scattering, the time when this light last bounced off of a nearby electron and just free-streamed out. After this time of 'free-streaming' the universe changed from being opaque to being transparent, the light could now get around.

This free-streaming light is the Cosmic Microwave Background radiation. The CMB is still free-streaming today and makes up almost all of the light in the universe. In fact, there are about 411 photons of this light in every cubic centimeter in the universe. Just like the early universe which they have traveled from, the CMB photons look *nearly* homogeneous and isotropic (though since we can only observe these photons from one place in space we one really care about it being isotropic or anisotropic). However, since there were very small perturbations in this primordial soup, the light in the CMB also has very small anisotropies, differences from the average value depending on where you look at it on the sky. The exact way in which these anisotropies appear is strongly dependent on the acoustic waves present in the early universe and thus on the details of the interactions of fundamental particles at these early times. Some good places to find more in depth coverage of these topics are the web pages of Dr. Wayne Hu (`http://background.uchicago.edu/index.html`) and Dr. Max Tegmark (`http://space.mit.edu/home/tegmark/movies.html`).

Due to this strong dependence, the CMB, along with other cosmological observables, has proven to be an excellent probe of the edges of our current understanding of the universe. This includes the physics surrounding the mysterious particles called neutrinos. Neutrinos are understood in the standard model of particle physics (SM) to be solely weakly ineracting particles. This means that they are electromagnetically neutral (they do not interact via the electromagnetic force) and they are massless, meaning that they travel at the speed of light. The reason exactly *why* neutrinos are massless in the standard model, becomes a very in-depth question that requires a deep background in quantum field theory (after of course an in depth background in group theory, differential geometry, and complex analysis). To avoid having to rewrite several courses worth of knowledge, the reader is asked to trust me that neutrinos are massless in the standard model. If the reader is interested in learning some of the physics associated with this topic the textbooks "*Modern Differential Geometry for Physicists*" by Chris Isham, "*Group Theory and its Application to Physical Problems*" by Morton Hammermesh, and "*Quantum Field Theory*" by Mark Srednicki. These are, of course, just some recommendations among several possible texts to consult.

1.2. Neutrinos Have Mass

The problem is that the standard model is incomplete. One of the most notable examples of this fact is that experimental evidence has shown that neutrinos must have mass (Abe et al. 2014). This has most distinctly been shown by large detector experiments in Japan and Canada which have worked in tandem to try to place limits on neutrino physics. In these experiments giant tanks of water placed hundreds of meters underground to avoid background noise and lined by photomultiplier tubes to amplify the signals of detections, were closely analyzed. What the observers were hoping to see was light created by the recoil of a water molecule when the nucleus was hit by a high-energy neutrino. As you might imagine since neutrinos interact solely via the Weak Nuclear Force, these experiments ran quite awhile before getting useful data. However, after enough collisions these detectors were able to tell that neutrinos had a tendency to 'oscillate' between the different flavors. By flavors here I am referring to either electron, muon, or tau neutrinos, the three known families of leptons (spin-1/2 particles). Due to again some more complicated physics, neutrinos could only osciallte between different flavors if the different flavors had different masses. Since there is only positive mass in the universe (as far as we know) this then implied that neutrinos (at least two of the three flavors) were massive, in contrast to the standard model!

So there must be some explanation for this. The current SM theory of neutrinos, their interactions with each other, and their interactions with the cosmos is limited to interactions through the weak nuclear force, as mentioned earlier. To get an idea of how neutrinos affect the cosmos in general let's first analyze the case of this interaction which we know exists. To do this, we will first need to define some terms to talk about the interaction. The first is the interaction rate, denoted by the symbol Γ, for a certain type of particle interaction, which is fairly self-explanatory. This is the rate, in units of time^{-1}, is the rate at which the interaction occurs, such as collisions per second for a collisional interaction. In the case of the neutrino weak interaction, the only thing that we are interested in is the collision of neturinos with other neutrinos. This is ignoring a considerable part of the number of interactions that neutrinos can undergo via the weak interaction, but it is the interaction which has the strongest effect on the part of the cosmos that we are interested in (the CMB) and it is the part that is directly analogous to the interactions that we will be talking about. Anyway, collisional interactions are dependent on the cross-section for the interaction, which is denoted by the symbol σ. This quantity is the area around a particle (orthogonal to the axis of collision of two particles) which another particle must pass through in order for their to be a 'significant' interaction. If you think that that sounds rather vague, you'd be right, and understanding the concept of a cross-section and how to calculate one for a given interaction is actually a very in-depth question. The calculation of this quantity requires a strong background in quantum field theory and is almost always a perturbative (non-exact) result. Finally, collisional interactions, and the reaction rate associated with them are also dependent, as one would expect, on the number density, usually denoted as n, of the particles in question. That is, the more closely packed a group of particles are, the more likely they are to interact with each other.

So how can we come to understand neutrinos in the context of cosmology with these terms? Well, as the interaction rate, Γ, of the neutrino scattering is proportional to the number density of neutrinos n_ν and cross section σ_ν of interactions between neutrinos, we are able to predict when neutrinos will effectively 'stop' interacting with eachother in the universe if we know these quantities as a function of time. This concept of the point when an interaction 'stops' is again a rather vague term but is roughly described to be the point in time when the chance for a random neutrino to have an interaction again in its life time is less than 1. Then, if we know σ_ν and n_ν we can predict this time of 'free-streaming,' when the neutrinos stop interacting with one another. Given the correct values it turns out that the weakness of the neutrino interaction via the weak force corresponds to a free-streaming of neutrinos in the very early universe, at $T \approx 1.5$MeV. Here we use temperature (in natural untis) as a substitute for time, something that cosmologists like to do a lot. This is because the 'temperature' of the universe (defined to be the temperature of the CMB as a blackboady spectrum) is a monotonically decreasing function of time, so that a single point in time corresponds to a unique temperature. As the temperature of the CMB today is on the order of milli electron volts, you can imagine that this was very early in the history of the universe. This free-streaming has a large impact on cosmological observables like the CMB that we can predict and measure, if the time of free streaming were to change this would have a notable impact on the cosmos.

However, we have many reasons to believe that this is not the full story. Specifically, there is strong evidence (Abe et al. 2014) that neutrinos have mass, however small it may be as we alluded to above. This observation, in contradiction to the SM theory of massless neutrinos, immeditely implies that neutrinos must have some type of interaction beyond the SM, and that this interaction must be mediated by particles which have not, as of yet, been observed. So, the question then becomes, *“Where could this mass possibly come from?"*

1.3. Models

Work from multiple authors (Cyr-Racine & Sigrudson 2014; Archidiacono & Hannestad 2014) has shown that current data allows for much flexibility in the underlying theoretical assumption of neutrinos interacting solely via the W and Z vector bosons and the weak nuclear force. Studies exploring this range of new non-standard neutrino interactions have spanned several different models. The plethora of available models to explain neutrino mass reflects how hard it is to understand neutrinos. However the large laboratory of the cosmos has allowed for strong constraints to be placed on a lot of these models. Ranging in application from supernovae (Manohar 1987) to the predictions of Big Bang Nucleosynthesis (Ahlgren et al. 2013), data has been able to effectively rule out many models.

Some of the more popular models that have been considered have included majoron-like models for neutrino interaction. The “majoron-like" qualifier here refers to the fact that in these theories neutrinos are considered to be their own anti-particles. These interactions are unique in their ability to be probed and ruled out by astrophysical data, such as those mentioned above as well as neutrinoless double β-decay and the decay width of the Z boson (Lessa & Peres 2007). These models are named for the famous Italian physicist Etrore Majorana, who first proposed such models. These models account for neutrino mass through coupling to a massive Goldstone boson, created through spontaneous $U(1)$ symmetry breaking. Most likely that was a lot of words you didn’t know all at once so lets slow down for a second. Without getting in to too much of the quantum field thoery (QFT) that is takes to truly explain the all of the above, I’ll just go over what some of the terms mean. In QFT a lot of the main concepts are explained in terms of the symmetries of a set of laws or of a system. If you know the set of symmetries of a set of laws or of a system, it can tell you a lot about how the system will evolve.

If a system in a high energy state obeys a certain symmetry (for example it is invariant under translations) and the laws governing the system (the Lagrangian or Hamiltonian of the system) obey that symmetry, but the low energy state of the system *does not* obey this symmetry we say that the symmetry is *spontaneously broken*. That is that as a system is ‘cooled’ from a high energy state to a low energy state the arragnement of the system undergoes a spontaneous change so that the system no longer has this symmetry, but the underlying laws of the system still have this symmetry. To give you an example, imagine that I have a lot of free, non-interacting particles that are in the high-energy state of a gas, like in the room around you. Now this system looks exactly the same if I move a few feet in any direction (okay not exactly, but if you have a really big room it looks basically the same) so that it is translationally invariant. The Hamiltonian of this system is also invariant under these translations, this would simply be the Hamiltonian of a lot of free particles:

$$H = \sum_{i=1}^{N} \frac{p_1^2}{2m}$$

However, if one were to cool the air in your room so that eventually it condensed, so cool even that it solidified in to a crystal lattice, the system would no longer have a continuous translational symmetry, but the underlying laws of the system still would. In this sense the property of a system having a certain symmetry would be 'broken' and the system would no longer be symmetric. This in particular would be the spontaneous breaking of a continuous translational symmetry (a translation in any direction by any distance). In the theory of Lie Groups, which are an important part of modern differential geometry, these set of continuous translations is a representation of the Lie Group $U(1)$, which is formally defined to be the set of all $A \in \mathbb{C}^{1\times 1}$ such that A is a unitary matrix. If you think of these matrices as just complex numbers then this si just the set of all complex numbers with magnitude 1, which is the circle in the complex plane. The reason why translations can be a representation of this group is rather clear from the similar way that the group $U(1)$ combines under multiplication and the way translations combine under composition.

So, back to how this relates to neutrinos. Most of the models that have sought to explain neutrino mass have done so by using a new theoretical interaction between neutrinos which is mediated by a massive Goldstone Boson. A Goldstone Boson is a type of spinless particle which arises in theories which have the spontaneous breaking of a continuous symmetry, such as the spontaneous breaking of a $U(1)$ symmetry, and are named for the physicist Jeffery Goldstone. These models are also popular because we believe that neutrinos do have a spontaneous breaking of this symmetry. To be clear, the exact reason why this symmetry breaking creates a Goldstone Boson and thus neutrino mass is not important for our purposes here. It is however, relevant that the reader understands the difference between these models and others when perhaps encountering any future reading.

Most approaches to comparing these models with data have accepted a phenomenological model via coupling to an effective scalar or pseudoscalar field which correspond well with the majoron-like models. These scalar and pseudo-scalar fields would be put in place to describe the presence of this new massive boson, which would mediate this intercation. The difference between scalars and pseudo-scalars are not important, especially for our purposes, as we will only be concerned with a simplified version of the scalar case, as is elaborated on below. These effective approaches often make a few other simplifying, but well justified (Oldengott et al. 2015; Archidiacono & Hannestad 2014), assumptions which further simplify the analysis. Most commonly, these assumptions involve only treating limiting cases of the mass of the corresponding new boson, which we will call ϕ. The limit of small or zero boson mass M_ϕ, while still phenomenologically important, presents multiple complicating factors. In order to simplify matters, we will consider here only the limit of an extremely massive mediating particle ϕ, which has been studied in the past by (Cyr-Racine & Sigrudson 2014) referred to throughout the rest of this work as CRS14. These studies have shown consistency with the data of interactions effectively 9 orders of magnitude stronger than the corresponding Fermi interaction of the electroweak force, (Archidiacono & Hannestad 2014) and CRS14. As these constraints are based purely on the CMB data they remain rather robust.

We begin by discussing the state of these models in depth and the different physical realities that they correspond to, focusing more on the model that we analyze in depth here. We then expand upon these past models by applying this type of heavy-mediator model to the latest CMB data available. Additionally, we use statistical analysis techniques novel to this study, including a Bayesian inference method called MultiNest, which is explained in depth in 3.1. These analyses are specifically suited to this type of task due to the multi-modal (meaning two or more different “best fit" scencarios) nature of the posterior distribution (probabilities on parameter values infered from analysis) that is prevalent in results associated with previous studies including CRS14. These novel techniques combined with wrapping the parameter inferecne and cosmological calculations with the new cosmological parameter estimation system CosmoSIS allow for quick convergence of the analysis, better posterior distributions due to multi-modal optimized code, and easy sharing and peer review of code as well as results through the CosmoSIS system (Zuntz et al. 2015). For a better background on the way in which the parameter analysis works the reader is referred to the numerous papers and explanations of the way in which this software works. These are available at the following links:

Program	**Link**
MultiNest	`https://ccpforge.cse.rl.ac.uk/gf/project/multinest/`
CosmoSIS	`https://bitbucket.org/joezuntz/cosmosis/wiki/Home`
CAMB	`http://camb.info/`

Finally, we use a Fisher Matrix analysis technique in order to predict how future studies may be able to put better constraints on the phenomenological model we test here. We use the type of analysis that is used in (Wu et al. 2014), and we elaborate on that model further here in section 3.2. As we will see, this analysis also offers further insight into the posterior distributions we present in this paper. This interpretation provides even stronger evidence for the strength of these results remaining robust in the future.

The rest of the paper may become quite technical and for some terms the reader is referred to Scott Dodelson's book, "*Modern Cosmology*," which gives an in depth review of the meaning of a Boltzmann Hierarchy (which we will reference later) and other important terms in the study of the Cosmic Microwave Background Radiation such as visibility, opacity, transfer functions, Legendre decomposition, and others. Unlike the previously mentioned texts, the reader is strongly encouraged to look through this book as it contains information that is key to understanding a project in this area. For a project specifically associated to this area of research (non-SM neutrinos) the reader should also look at CRS14 and (Archidiacono & Hannestad 2014), and for further mathematical reference (Oldengott et al. 2015) and (Ma & Bertschinger 1995).

2. Interaction Models

When approaching the problem of how to test and look for neutrino interactions beyond the standard model of physics, phenomenological approaches that have been taken in the past most readily start by accepting a simplified Lagrangian density including both scalar and pseudoscalar interactions, for example as shown below in equation 1. In order to treat these cases simplifications are usually implemented in treating the scalar and pseudoscalar interactions separately, as well as taking limits on the mass of the mediating boson, referred to here as ϕ.

These models can actually be very representative of the effects that non-SM neutrino interactions could have as a whole as they can be, dependent on your choice of simplifications, mapped to either a late time *recoupling* (non-interacting neutrinos suddenly starting to interact again) or *decoupling* (early-time interacting neutrinos no longer interacting) of neutrinos, the two effects that we are able to put quantitative constraints on within the context of cosmology. As is describeed in (Archidiacono & Hannestad 2014), we may view the scalar and pseudoscalar cases as limiting cases of a neutrino interaction mediated by a gauge boson of finite mass M_ϕ. At high temperatures or energy this will have the same temperature scaling dependence of the massless mediator. At low temperatures or energy we may view the massive scalar case we explore here as an effective field theoretic approach, resulting in a Fermi-like interaction. What matter for the readeer is that this model results in neutrinos acting very simliarly to the way that they do in the standard model except much more strongly. In this way we may see that though there are many generalizations to this model, we are able to effectively characterize the interactions that we are able to put constraints on with only these.

Similarly, as discussed in (Oldengott et al. 2015) we may also view these models as limiting cases on the mass of the mediator boson ϕ (rather than as limiting cases of the energy of the theory) and the associated coupling constants described by the same Lagrangian density:

$$\mathcal{L}_{\rm int} = \mathfrak{g}_{ij}\overline{\nu}_i\nu_j\phi + \mathfrak{h}_{ij}\overline{\nu}_i\gamma_5\nu_j\phi \tag{1}$$

Where above, $\mathfrak{g}_{ij}$ and $\mathfrak{h}_{ij}$ are scalar and pseudo-scalar coupling constants. As stated above, in this study we only consider the effects of scalar interactions, setting $\mathfrak{h}_{ij} = 0$ and assume flavour-independent, diagonal coupling so that $\mathfrak{g}_{ij} = 0$ for $i \neq j$, as is the case in the derivations given by (Oldengott et al. 2015). This means that we do not consider the tau neutrino, mu neutrino, or electron neutrino to interact with each other, but rather only within the smae generation. That is electron neutrinos only interact with electron neutrinos and so on. In this way we consider no pseudoscalar interactions and are left with the simple Lagrangian density of:

$$\mathcal{L}_{\rm int} = \mathfrak{g}_{ii}\overline{\nu}_i\nu_i\phi \tag{2}$$

2.1. Massive Mediator

As discussed above we test in depth in this paper the case of neutrinos self-interacting via a massive scalar ϕ, with corresponding mass M_ϕ. This interaction is then described by a coupling to this mediator particle with dimensionless effective coupling strength $g_\nu = \mathfrak{g}_{ii}$. As we assume the mass of the mediator to be very large we may assume that at all temperatures of interest the temperature of the neutrinos is significantly less than the mediator mass. With this assumption in mind we may ignore the interactions of this scalar boson with each other and with the neutrinos and treat the interaction only occuring between neutrinos in the same family. That is, while the boson *mediates* the interaction, we do not have to account for the scattering that it undergoes itself having any significant impact on the cosmos. We can then characterize the interaction with the following *dimensionful* coupling constant.

$$G_{eff} \equiv \frac{g_\nu^2}{M_\phi^2} \tag{3}$$

In this study we vary only the strength of this effective coupling constant for constraints. Though it is clear from equation (3) that there is a degeneracy between the coupling constant g_ν and the mediator mass M_ϕ, which could be varied jointly in a future study. As it pertains to this study, we assume throughout the parameter space that the mediator is of significant mass, corresponding only to larger g_ν at large G_{eff}, equivalent to basically fixing M_ϕ at a large value in the parameter variation. We note that as relatively large values of G_{eff} are the only values of significant physical interest, we take the assumption that $G_eff \gg G_F$, and then justifiably ignore the electroweak contribution to the way taht neutrinos interact with one another. This will most notably cme up in the way we talk about the neutrino opacity and visibility.

We may see that the neutrino opacity of this interaction is given by:

$$\dot{\tau}_\nu = -aG_{eff}^2 T_\nu^5 \tag{4}$$

Using this we can see that the product of the cross section for this interaction $\sigma_{\nu\nu}$ with the neutrino velocity v_ν, when thermally averaged, scales as $\langle\sigma_{\nu\nu}v_\nu\rangle \propto G_{eff}^2 T_\nu^2$, where T_ν is of course the neutrino temperature. This comes from the heuristic argument that: $\langle\sigma_{\nu\nu}v_\nu\rangle \propto \frac{\dot{\tau}_\nu}{n_\nu} \propto \frac{\dot{\tau}_\nu}{T_\nu^3} \propto G_{eff}^2 T_\nu^2$. If we move from here to define the visibility function for neutrinos in the usual way we have:

$$g_\nu(z) \equiv -\dot{\tau}_\nu e^{-\tau_\nu} \tag{5}$$

Clearly, for significiant values of G_{eff} the era of neutrino free-streaming may be significantly postponed as shown and visualized in CRS14.

Finally, for this phenomenological model, some may insist that there must be a separate account of the thermal history for neutrinos based on possible reheating of the neutrinos from annihilation of this new scalar particle. We insist that, at least for the purposes of this model, and for all times of which reheating would be of interest (i.e. after photon decoupling), the temperature of the neutrinos has cooled to such a state where significant production of the scalar particle ϕ can be neglected. We therefore accept the standard scaling of the neutrino temperature as $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{CMB}$.

2.2. Massless Mediator

Here we briefly discuss the case of the massless limit of the scalar boson in the Langrangian density given by equation (2). We note that the heroic calculation of the full derivation for the Boltzmann Hierarchy of both of these models is given in (Oldengott et al. 2015), however there are significant complications in treating the case of a massless mediator.

First we may note that, as in the case of the light pseudoscalar interaction considered by (Archidiacono & Hannestad 2014), the limit of a light scalar mediator here can be viewed phenomenologically as corresponding to neutrinos which decoupling from their electroweak interaction at early times $T_\nu \sim 1$ MeV, and then *recoupling* at later times, once the interaction rate overtakes the cosmic expansion rate. This is because the interaction rate for this limit is given by $\Gamma_{\text{massless}} \sim g_\nu T_\nu$, where g_ν and T_ν are the same as described in the massive case. In this way, due to the scaling of the interaction rate with the neutrino temperature they are able to recouple to each other.

There are fundmental difficulties in approaching this model in the same way that we do the massive case. The greatest of these is the fact that the annihilation interaction $\nu\nu \rightarrow \phi\phi$, which can be ruled out as thermally impossible at all epochs of interest in the case of a massive mediator boson, now becomes not only possible but likely. This calls for an account of the Boltzmann Hierarchy of the new boson ϕ, and considerations of all other interactions beyond simply the $\nu\nu \rightarrow \nu\nu$ scattering process. Until there exists suitable evidence to suggest the validity of this sort of model, implementation of this treatment in the massless case will likely not be seen in the near future.

There do exist considerable simplifications to this model which admit simple implementations. One such example is that implemented by (Archidiacono & Hannestad 2014) in which they describe this interaction by varying a redshift of recoupling $z_{\rm re}$, at which neutrinos switch from being not coupled, to being infinitely strongly coupled (suppression of all anisotropic stress terms). They draw a correspondence between this "turn-on" redshift and the dimensionless coupling constant g_ν and there by use constraints on $z_{\rm re}$ to place constraints on the coupling strength, considering both diagonal and off-diagonal coupling.

However, we caution uses of simplifications of this kind as the same type of model given in (Archidiacono & Hannestad 2014) for the massive case lead to constraints on the coupling strength which are contradictory to our results. This then may be too much of an oversimplification.

3. Data Analysis Methods

3.1. MultiNest

We use the software package MultiNest (Feroz & Hobson 2008; Feroz et al. 2009) to scan the parameter space and generate random samplings of the n-dimensional posterior PDF. MultiNest uses a nested-sampling algorithm to evolve a set of ‘live points’ that are initially distributed according to the prior. Iteratively, the point with lowest likelihood is replaced by a new point that is drawn from the prior. The set of live points gradually becomes confined within regions of higher likelihood and smaller volume, enabling the Bayesian ‘evidence’ to be computed with increasing accuracy. The algorithm terminates when the accuracy reaches a specified threshold; following the recommendation in MultiNest’s documentation, we adopt a tolerance of 0.05 in log-evidence, with sampling efficiency of 0.1. As a by-product of this calculation, MultiNest returns a random sampling from the posterior PDF.

For a given parameter, X, we then calculate moments of the marginalized, 1D posterior PDF as follows. The first moment is the mean, $\overline{X} \equiv N^{-1}\sum_{i=1}^{N} x_i$, which we take to be the central value. The second moment is the variance, $\sigma_X^2 \equiv (N-1)^{-1}\sum_{i=1}^{N}(x_i-\overline{X})^2$, which we take to be the square

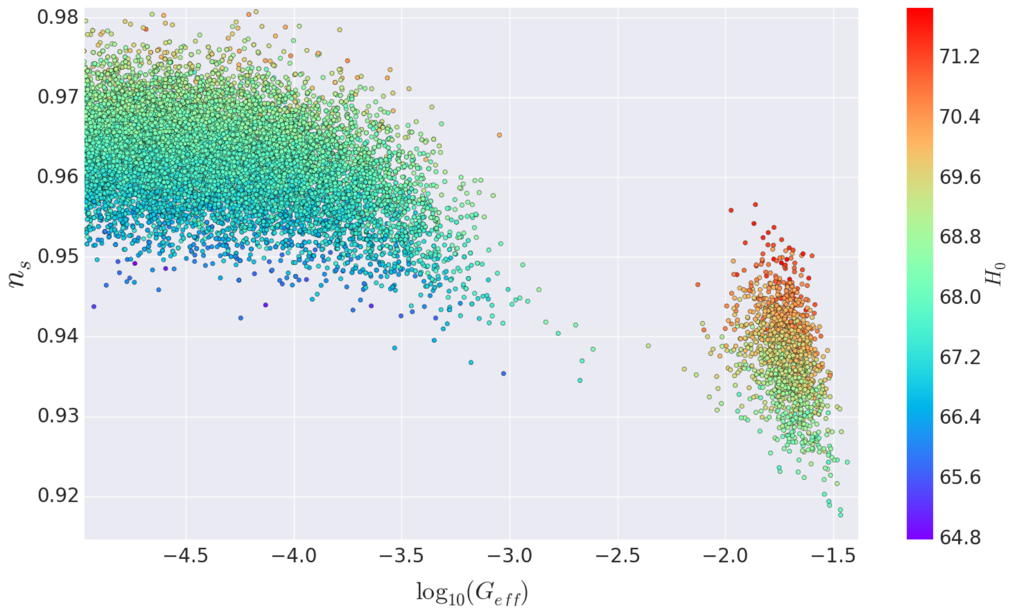

Fig. 1.— We show points randomly sampled from the posterior probability distribution created by MultiNest for contraints including the low and high-l temperature and polarization power spectra. These points clearly demonstrate the degeneracy between the neutrino interaction and the values of the parameters for scalar perturbations as well as the favoring of higher values of H_0 for these interactions.

of the 1σ credibility interval.

In this Bayesian Inference analysis we use multiple contraining data sets to define the likelihood over the parameter space including the Planck 2015 Data 'lite' likelihoods for temperature and polarization, just temperature, and for `low_l` multipoles which we will refer to as `TTTEEE, TT, low_l` respectively (Planck Collaboration et al. 2015b). We also use constraints in the size and shape of the matter power spectrum from SDSS-III (Chuang et al. 2013), referred to as `BAO`. As the 'lite' likelihoods which we employ already marginalize over the nuisance parameters as described in `CITE`, we do not employ the use of the SPT and ACT data sets for higher multipoles, which mainly contribute in that they put constraints on these nuisance parameters.

In light of the fact that we are considering data which is already marinalized over the nuisance parameters, we restrict our parameter space to the seven values represented by the parameter vector $\boldsymbol{\theta} = \{A_s, n_s, \tau, H_0, \Omega_b h^2, \Omega_c h^2, \log_{10}\left(G_{eff}\ \mathrm{MeV}^2\right)\}$. We then accept the flat priors for the standard cosmological parameters as described in (Planck Collaboration et al. 2015b) as well as a flat prior on $\log_{10}\left(G_{eff}\ \mathrm{MeV}^2\right) \in [-5.0, 0.0]$.

3.2. Fisher Matrix Analysis

Here we use the same approach as taken in (Wu et al. 2014) as well as others. This method is a simplified way to approximate the ability of future data to put constraints on values of parameters in a cosmological parameter space of particular interest. This type of analysis implicitly assumes that the distribution of the PDF in parameter space for this model is gaussian. We can immediately note that this is perhaps not the bext assumption for the PDF we find for $\log_{10}\left(G_{eff}\mathrm{MeV}^2\right)$, however the mode of scientific interest in this study, the interacting mode, can be seen to be well approximated by a gaussian distribution, so that this treatment is still well justified.

Many approaches have been taken to try to account for the fact that the Fisher approxiamtion might not represent the actual distribution in parameter space (Wolz et al. 2012; Perotto et al. 2006). Yet for our purposes here, especially with a large parameter space, it is computationally efficient to use the Fisher Matrix approach to the estimation of future constraints on the data.

Parameters	Fiducial Values
$\Omega_\mathrm{b}h^2$	0.02225
$\Omega_\mathrm{c}h^2$	0.1198
τ	0.079
H_0	67.27
n_s	0.9645
$10^9 A_\mathrm{s}$	2.207
$10^2 G_\mathrm{eff}\ \mathrm{MeV}^2$	1.0

Table 1: Fiducial parameter values used to take partial derivatives about in order to treat the Fisher Matrix Analysis. We choose the best fit parameters of (Planck Collaboration et al. 2015a) for the standard cosmological parameters and then choose the mean value of the interacting mode given by CRS14 in order to explore future constraints on this interacting mode.

So, assuming that the the likelihood $\mathcal{L}$ of the parameters vector $\boldsymbol{\theta}$ given the vector of data $\mathbf{d}$ to be Gaussian we have (Wu et al. 2014):

$$\mathcal{L}(\boldsymbol{\theta}|\mathbf{d}) \propto \frac{1}{\sqrt{|\mathbf{C}(\boldsymbol{\theta})|}} \exp\left(-\frac{1}{2}\mathbf{d}^\dagger \left[\mathbf{C}(\boldsymbol{\theta})\right]^{-1} \mathbf{d}\right) \tag{6}$$

Where $\mathbf{C}$ is the covariance matrix associated with the data, which we will define further below, and $\mathbf{d}$ is the vector $\mathbf{d} = \{a_{lm}^T, a_{lm}^E, a_{lm}^d\}$ of the spherical harmonic coefficients for the temperature, E mode, and lensing-indused deflection respectively. The vector $\boldsymbol{\theta}$, then contains the parameters of interest (no nuisance parameters) of $\boldsymbol{\theta} = \{A_s, n_s, \tau, H_0, \Omega_b h^2, \Omega_c h^2, \log_{10}\left(G_{eff}\mathrm{MeV}^2\right)\}$. The Fisher matrix itself is then built by taking numerical partial derivatives of the likelihood, or its curvature, at the fiducal values of the parameters$\boldsymbol{\theta}_0$:

$$F_{ij} = \left\langle \left. \frac{\partial^2 \log \mathcal{L}}{\partial\theta_i \partial\theta_j} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \right\rangle \tag{7}$$

Combining the above equations (7) and (6) we see that we are able to state the elements of the Fisher Matrix as:

$$F_{ij} = \sum_l \frac{2l+1}{2} f_{sky} \mathrm{Tr}\left(\mathbf{C}_l^{-1}(\boldsymbol{\theta}) \frac{\partial \mathbf{C}_l}{\partial\theta_i} \mathbf{C}_l^{-1}(\boldsymbol{\theta}) \frac{\partial \mathbf{C}_l}{\partial\theta_j} \right) \tag{8}$$

Where l is the multipole of the spectra and $\mathbf{C}$ is the covariance matrix, mentioned above, and defined by:

$$\mathbf{C}_l \equiv \begin{pmatrix} C_l^{TT} + N_l^{TT} & C_l^{TE} & C_l^{Td} \\ C_l^{TE} & C_l^{EE} + N_l^{EE} & 0 \\ C_l^{Td} & 0 & C_l^{dd} + N_l^{dd} \end{pmatrix}$$

Where the power spectra include the Gaussian noise N_l^{XX} defined as:

$$N_l^{XX} = s^2 \exp\left(l(l+1) \frac{\theta_{\mathrm{FWHM}}^2}{8 \log 2} \right) \tag{9}$$

This is exactly parallel to (Wu et al. 2014) in that we do not consider a B-Mode power spectrum signal, ignoring the lensing of the polarization and that there are no primordial B-modes arising from inflation. We set $C_l^{Ed} = 0$ as it has a very small effect.

We then obtain 1-σ uncertainties for the parameters by marginalizing over the other parameters. For example for the parameter θ_i we have:

$$\sigma_i \equiv \sigma(\theta_i) = \sqrt{(\mathbf{F}^{-1})_{ii}} \tag{10}$$

We then use these 1-σ estimates for comparison with our current results as well as for prediction of the abilities of future results to give constraints on interactions of this type. As is implied by the above, we must accept for this analysis a set of fiducial parameters $\boldsymbol{\theta}_0$ which are given in table **??**. We note that we intentionally accept a high-value for G_{eff}, consistant with the interacting neutrino mode, so that we are better able to explore the future constraints on the interesting

Parameters	Standard Mode	Interacting-ν Mode	1-σ Planck	1-σ CMB-S4
$\Omega_b h^2$	0.0222 ± 0.0002	0.0225 ± 0.0002	0.00014	0.00003
$\Omega_c h^2$	0.119 ± 0.002	0.120 ± 0.002	0.0014	0.0005
τ	0.10 ± 0.02	0.10 ± 0.02	0.007	0.004
H_0	67.9 ± 0.7	69.6 ± 0.7	0.6	0.2
n_s	0.962 ± 0.006	0.938 ± 0.005	0.004	0.002
$10^9 A_s$	2.27 ± 0.10	2.16 ± 0.09	0.0327	0.015
$\log_{10}(G_{\rm eff}\mathrm{MeV}^2)$	$< -3.5\,(95\%\,\mathrm{C.L.})$	-1.7 ± 0.1	N/A	N/A
$10^2 G_{\rm eff}\ \mathrm{MeV}^2$	$< 0.03\,(95\%\,\mathrm{C.L.})$	2.0 ± 0.4	0.3	0.04

Table 2: Marginalized constraints on cosmological parameters for the two main modes of the distribution. Unless otherwise indicated, we quote 68% confidence level for Planck + BAO.

physics of this mode as well as the fact that the posterior of this mode is well fit by a gaussian, which tailors to the current treatment. We also take partial derivatives over the value $G_{\rm eff}$ rather than $\log_{10}\left(G_{\rm eff}\mathrm{MeV}^2\right)$, as this is better suited to the linear approxiamtion of the Fisher Matrix.

4. Results

We review our results in Table 2 where it can be easily seen that we get excellent agreement between the constraints placed on the parameters by the MultiNest Analysis and those predicted by the linearized approximation of the Fisher Matrix Analysis. We note that, because we are accepting a fiducial parameter about the interacting mode, the 1-σ given for this Fisher Matrix treatment cannot be reasonably applied to the non-interacting mode.

Acknowledgements

The work of this paper could not have been completed without the advice and review provided by members of the Knox research group: Dr. Marius Millea, Zhen Pan, and Brigid Mulroe. We would especially like to thank Dr. Joe Zuntz, leader of the CosmoSIS team, for his excellent and quick feedback on technical issues, without which this study could not have been completed in time. We thank Shadab Alam of the Carnegie Mellon University McWilliams Center for Cosmology for his work in putting together the necessary materials for a likelihood comparison with the BOSS DR11 data set. We would like to thank the McWilliams Center for Cosmology, for the use of their ‘Coma Cluster’ computer cluster, on which the computational work for this paper was run.

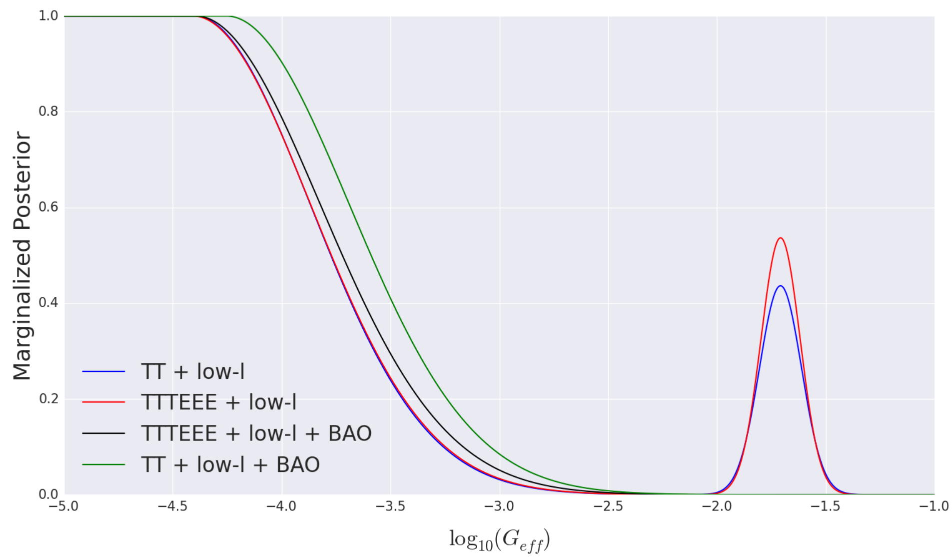

Fig. 2.— This shows the 1D-PDF marginalized to show the probability distribution on the value of $\log_{10}\left(G_{eff}\mathrm{Mev}^2\right)$. This uses constraining data from the Planck 2015 'lite' likelihood for both the temperature and polarization power spectrums, the low-l data (Planck Collaboration et al. 2015a), and results from the SDSS DR11 data set `citation needed`. The distributions are smoothed by fitting the sum of two gaussians, one with a constant value below the maximum, to the binned marginalized data. The red line includes constraints from the temperature and polarization power spectra while that blue line contains constraints only from the temperature power spectrum.

The contents of this work were supported in part by the NSF grant PHY-1263201 for Research Experience for Undergraduates (REU) and was carried out in the hospitality of the University of California, Davis.

REFERENCES

Abe, K., Adam, J., Aihara, H., Akiri, T., Andreopoulos, C., Aoki, S., Ariga, A., Ariga, T., Assylbekov, S., Autiero, D., & et al. 2014, Physical Review Letters, 112, 061802

Ahlgren, B., Ohlsson, T., & Zhou, S. 2013, Physical Review Letters, 111, 199001

Archidiacono, M., & Hannestad, S. 2014, Journal of Cosmology and Astroparticle Physics, 7, 46

Chuang, C.-H., Prada, F., Cuesta, A. J., Eisenstein, D. J., Kazin, E., Padmanabhan, N., Sánchez, A. G., Xu, X., Beutler, F., Manera, M., Schlegel, D. J., Schneider, D. P., Weinberg, D. H., Brinkmann, J., Brownstein, J. R., & Thomas, D. 2013, MNRAS, 433, 3559

Cyr-Racine, F., & Sigrudson, K. 2014, AJ, 109, 1071

Feroz, F., & Hobson, M. P. 2008, MNRAS, 384, 449

Feroz, F., Hobson, M. P., & Bridges, M. 2009, MNRAS, 398, 1601

Lessa, A. P., & Peres, O. L. G. 2007, Phys. Rev. D, 75, 094001

Ma, C.-P., & Bertschinger, E. 1995, ApJ, 455, 7

Manohar, A. 1987, Phys. Lett., B192, 217

Oldengott, I. M., Rampf, C., & Wong, Y. Y. Y. 2015, Journal of Cosmology and Astroparticle Physics, 4, 16

Perotto, L., Lesgourgues, J., Hannestad, S., Tu, H., & Y Y Wong, Y. 2006, Journal of Cosmology and Astroparticle Physics, 10, 13

Planck Collaboration, Ade, P. A. R., Aghanim, N., Arnaud, M., Ashdown, M., Aumont, J., Baccigalupi, C., Banday, A. J., Barreiro, R. B., Bartlett, J. G., & et al. 2015a, ArXiv e-prints

Planck Collaboration, Aghanim, N., Arnaud, M., Ashdown, M., Aumont, J., Baccigalupi, C., Banday, A. J., Barreiro, R. B., Bartlett, J. G., Bartolo, N., & et al. 2015b, ArXiv e-prints

Wolz, L., Kilbinger, M., Weller, J., & Giannantonio, T. 2012, Journal of Cosmology and Astroparticle Physics, 1209, 009

Wu, W. L. K., Errard, J., Dvorkin, C., Kuo, C. L., Lee, A. T., McDonald, P., Slosar, A., & Zahn, O. 2014, ApJ, 788, 138

Zuntz, J., Paterno, M., Jennings, E., Rudd, D., Manzotti, A., Dodelson, S., Bridle, S., Sehrish, S., & Kowalkowski, J. 2015, Astronomy and Computing, 12, 45