Kelvin Wave Oscillations on Vortices in Superfluid Helium

Ryan Gnabasik
Dr.Rena Zieve
UC Davis 2014 Physics REU

Current methods used for studying vortex phenomena are too slow at acquiring circulation values to be useful in investigating higher harmonic kelvin wave oscillations. Therefore, the development of new technique is necessary. This new technique still employs a vibrating wire to measure vortices, but now measures in fourier space. To check that this newly developed technique was plausible, frequency sweeps were done with the expectation of measuring a double lorentzian where the peak width changed based on the total amount of circulation around the wire. This was found to be the case for differing amounts of constant circulation around the wire. Vortex precession around the cell was also measured using the new technique leading to the conclusion that this technique is a viable option.

I. INTRODUCTION

A. Vortices and Superfluid

The study of vortices in superfluid helium provides an understanding of a common phenomenon. Anytime there is rotation in fluid flow, a vortex can exist. To characterize vortices, the circulation parameter is used. Circulation is defined by,

$$\kappa = \oint v \cdot dl, \tag{1.1}$$

where κ represents circulation and is equal to the velocity tangential to a closed loop. Because vortices can occur anytime there is fluid flow, the motivation to study them can be either pure or applied. The applications to other fields include aerospace engineering when the designing aerial vehicles. Meteorologists observe vortices in the form of tornadoes and hurricanes and astrophysicists consider vortices when looking at the rotation rate of neutron stars.

Superfluid helium is used to study vortices. A superfluid is a fluid that has zero viscosity or no resistance to flow much like a superconductor has zero resistance. The electrons in a superconductor can be treated as a superfluid. For helium to become superfluid it first needs to be a liquid. Helium 4 liquefies at 4.2 kelvin and helium 3 at 3 kelvin. The superfluid transition temperature is 2 kelvin for helium 4 and 1 millikelvin for helium 3.[1] Along with having zero viscosity, superfluid helium is also a great thermal conductor and is practically incompressible. Although helium is expensive and the temperatures required to be a superfluid are difficult to achieve the circulation parameter that characterizes vortices is quantized in helium 4 like so,[1]

$$\kappa = \frac{hn}{m_{He}}, \tag{1.2}$$

where h is Planck's constant, n is an integer, and m_{He} is the mass of the helium 4. Due to the quantization of the circulation, the vortex will not speed up or slow down and the vortex core does not grow or shrink.[2] As a consequence of quantization, all vortices in superfluid helium must have a core with the core not consisting of the superfluid.[1] It then follows that the vortices are easier to study in superfluids than normal fluids. By understanding vortices in superfluid helium in a quantized system, a basis is constructed from which other scientists can draw upon when considering vortices in normal fluid.

B. Fridge

To produce vortices in superfluid helium we first have to cool the helium down past its superfluid transition temperature. To do this we use a sorbtion-pumped helium-3 refrigerator. A diagram of the fridge can be seen in Figure 1. First, the fridge is lowered into a dewar of liquid helium 4 at its boiling point of 4.2 kelvin. Once this cools everything off, helium 4 is syphoned up into the fridge and pumped on. Due to evaporative cooling the helium 4 cools down to 1.5 kelvin. The helium 3 in a closed system can now condense. A charcoal pump inside the helium 3 system then pumps on it cooling it down to 300 millikelvin. The charcoal has a large surface area which pulls on the liquid and again employs the evaporative cooling concept. The helium 3 is in a closed system because it is expensive and rare. At the bottom of the fridge is a cell which is filled with helium 4 from a gas cylinder. This helium 4 is cooled down to 300 millikelvin from the closed system of helium 3. The helium 4 in the cell is cooled well below the superfluid transition temperature of 2 kelvin because the signal to noise ratio is much better. Spanning the length of the cell is a superconducting wire and on the outside of the cell are two superconducting magnets which together can produce a constant horizontal magnetic field in any direction depending on the amounts of current put in them. All of the superconducting material is cooled to the temperature of the helium 3.[1]

Production of the vortices involves rotating the fridge. This is accomplished by unplugging all of the wiring attached to the fridge and having the external pump for the helium 4 attached to a valve that allows for rotation. Although the superfluid has zero viscosity there is a maximum critical velocity where the superfluid will transition back to normal fluid.[1] This velocity is small and is easily achieved by rotating the fridge. Once the fluid is moving slower than the critical velocity it transitions back to a superfluid, but now contains vortices.

II. MEASUREMENT TECHNIQUES

A. Pulsed Mode

The superconducting wire that spans the length of the cell is used to detect vortices in the cell. If a current is sent through the wire which lies perpendicular to the constant magnetic field, the wire feels a Lorentz force and begins to vibrate much like being plucked. A vibrating wire in a magnetic field will then generate an emf which can be measured. In this system, it is more energetically favorable for the core of the vortex to be on the wire rather than off the wire.[1,3] However, once there is circulation around the wire then it begins to precess vibrating perpendicular then parallel to the magnetic field then back to perpendicular. This results from the wire feeling a force due to Bernoulli's law as can be seen in Figure 2. When the wire moves up, the helium rushes past it. Circulation around the wire from the vortex also contributes to the fluid velocity. If the circulation is counter-clockwise then when the wire is moving up the velocities add on the left side of the wire and subtract on the right side of the wire. Then due to Bernoulli's law, a higher velocity creates a lower pressure and as a result the wire experiences a force to the left. This is while the wire is still moving up and therefore the wire is pulled a small amount to the left. When looking at the wire moving down, the helium is rushing up now, and the velocities add on the right. This generates a force to the right again due to Bernoulli's law, pulling the wire to the right while it is still moving down. This precession effect moves the plane the wire vibrates in in the direction of circulation.[3]

Due to the precession of the wire vibrating parallel and perpendicular to the magnetic field, an envelope shape is produced that is seen in Figure 3. This envelope is then fit to the following,

$$|e^{-at} sin(\omega_o t) cos(d\omega t)|. \tag{2.1}$$

One circulation value is acquired from this and as seen in Figure 3, the oscillation takes around five to ten seconds to decay. An increased amount of circulation around the wire leads to more oscillations in the envelope. The $d\omega$ value can be converted to circulation. A single circulation value every five to ten seconds is adequate for studying many different types of vortex behavior, but not everything.

There are a few reasons why equation 2.1 is an exponential multiplied by two separate trig functions with

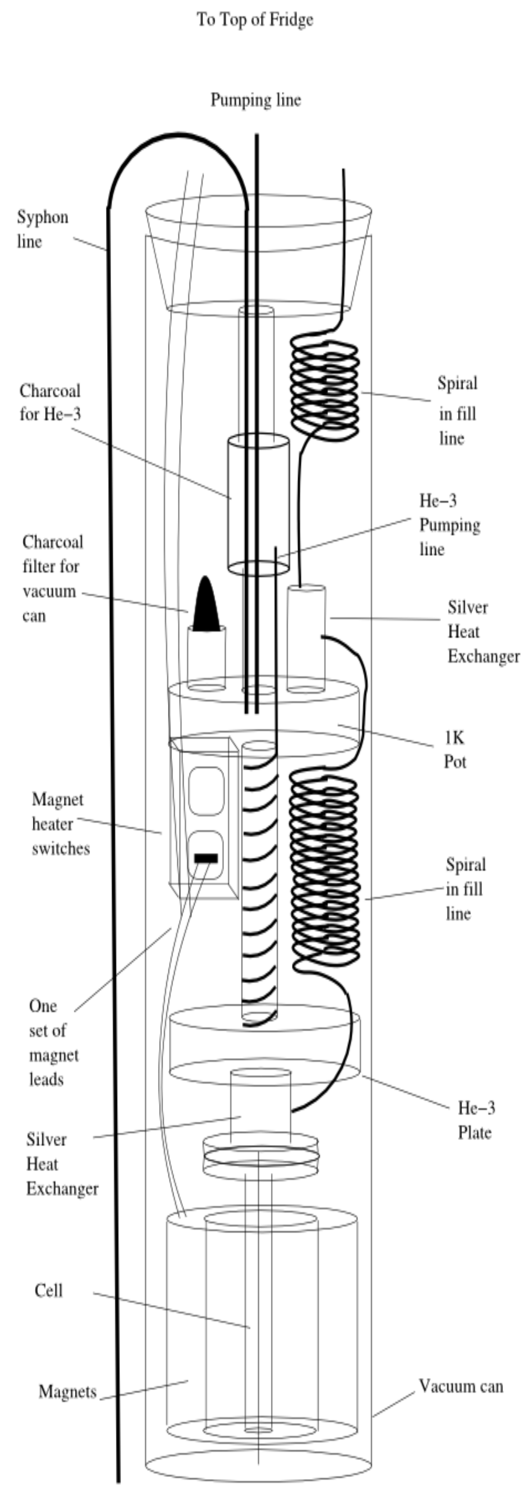

FIG. 1. A diagram of the fridge labeling major components. The syphon line is on the left which takes in the helium 4 from the dewar to be pumped on. The cell can be seen at the bottom of the fridge with the two magnets designated, inner and outer surrounding it. The superconducting wire also spans the length of the cell. The fridge is long and skinny because it needs to pass through the neck of the helium 4 dewar.[1]

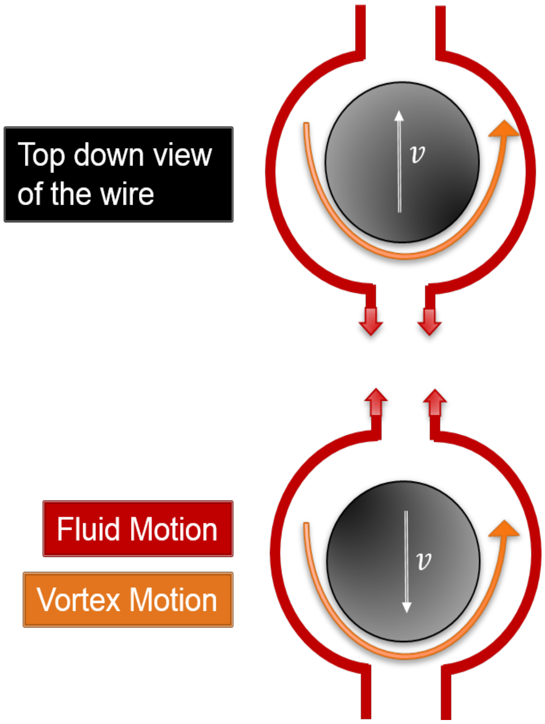

FIG. 2. A top down view of a cross-section of superconducting wire. When the wire moves down and the circulation is counter-clockwise the velocity of the helium is greater on the left side than the right side. The opposite is true for when the wire is moving up, but with the same direction of circulation. The wire feels a force in the direction of the side with higher velocity due to Bernoulli's law and will precess in the direction of vortex circulation.

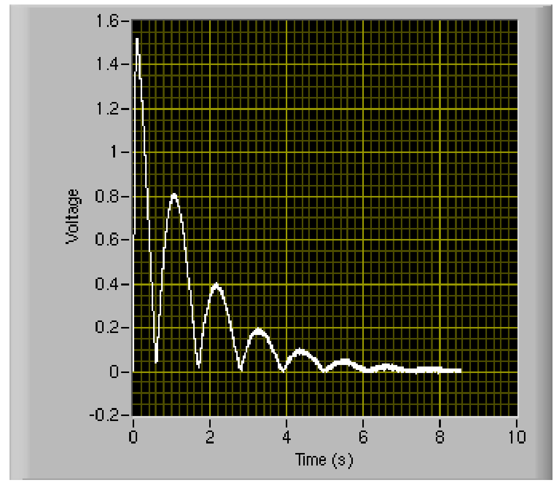

FIG. 3. The envelope structure is produced by the precession of the vibrating wire in and out of the magnetic field. At the maximum points the wire is vibrating perpendicular to the magnetic field and generates the full amount of emf. The minimums are when it vibrates parallel to the magnetic field. The decay time of the oscillations decreases with higher temperature. The envelope is fit using equation 2.1 to attain a single circulation value.

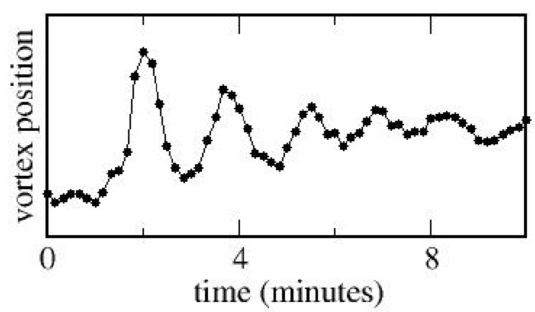

FIG. 4. Lowest harmonic kelvin wave oscillation with a period around 100 seconds. Because this kelvin wave is the lowest harmonic the period is long enough to use the pulsed mode technique to measure it. In this particular cell, there is a bump on one side that can easily catch vortices. The lowest order kelvin waves have a period around 42 seconds while pinned to the bump.[4]

different resonant frequencies. The simplest reason for a splitting in the resonance can be attributed to imperfections in the wire.[1] The second reason involves circulation around the wire. Without circulation the wire has two linear normal modes of vibration that are perpendicular to each other. Because circulation causes precession of the wire, those modes become circular. One is clockwise and the other counter clockwise. Each of these modes will have different frequencies due to the circulation of the vortex rotating in the direction of one of the modes and increasing the frequency.

B. Kelvin Waves

A kelvin wave is a perturbation along a vortex. When the wire is partially covered by a vortex, the vortex must continue to span from the wire to the cell wall. This section of the vortex is called a free vortex and the core consists of helium. The kelvin wave perturbations form along this free vortex when a pulse of current is sent through the wire, effectively plucking it. The free vortex end attached to the cell can remain stationary like a node, and the end attached to the superconducting wire can act like an anti-node. This is critical because then the total amount of circulation around the wire is changing and so the kelvin wave oscillations can be measured by looking at the circulation values if the previous behavior occurs. Figure 4 shows a kelvin wave oscillation with a period around 100 seconds. To study the kelvin wave oscillations, a vortex that has covered half of the wire and has the free vortex pinned to the cell wall is ideal. It is easier to pin vortices on large bumps on the cell wall which change the distance to the superconducting wire. The first harmonic has a period of 42 seconds while pinned to a large bump on one of the cells.[3] Any higher order modes have much shorter periods. The higher order kelvin waves exhibit interesting non-linear behavior that

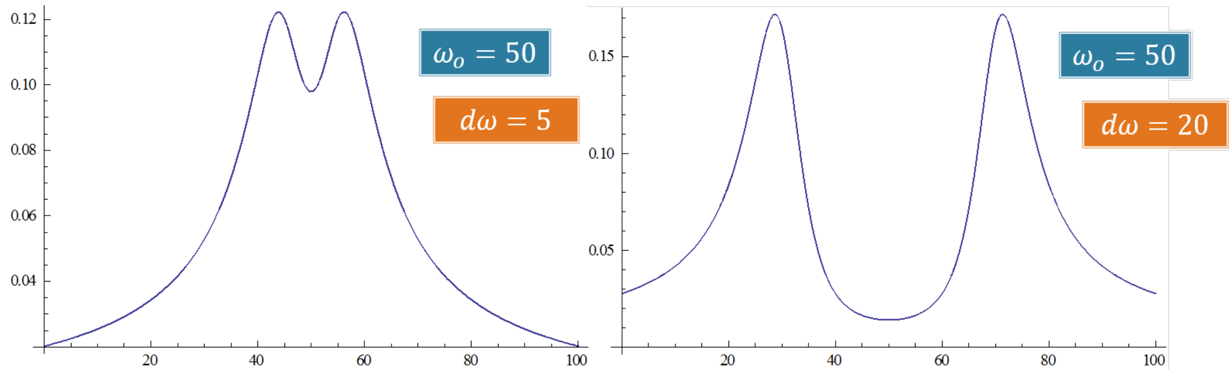

FIG. 5. The double lorentzian is the fourier transform of the response from the pulsed mode technique and therefore the expected response in the continuous method. ω_o parameterizes the center of the peaks while $d\omega$ determines peak separation.

is not completely understood. Because the pulsed mode technique can only acquire a circulation value every five to ten seconds, the amount of points that can be acquired during one period of a higher order harmonic kelvin wave becomes less than reasonable. Therefore the development of a technique that can acquire circulation values at a faster rate is necessary to study higher order kelvin wave oscillations.

C. Continiuous Mode

The solution to acquiring circulation values faster is to measure in fourier space. The original signal sent in for the pulsed mode can be represented by a dirac delta, $\delta(t)$. The fourier transform of a pulse is simply a sine wave at a constant frequency, $sin(\omega t)$. The response from the pulsed mode technique is represented by equation 2.1. Therefore the expected response should be the fourier transform of 2.1 which is a double lorentzian and is given by,

$$\frac{[a^3 + a(\omega - \omega_o)^2 + a(d\omega)^2]^2 + [a^2(\omega - \omega_o) + (\omega - \omega_o)^3]^2}{[a^2 + (\omega - \omega_o + d\omega)^2][a^2 + (\omega - \omega_o - d\omega)^2]}. \tag{2.2}$$

The ω_o value is where the double lorentzian is centered and the $d\omega$ is the separation distance between the two peaks which can be seen in Figure 5. Each peak represents one of the two resonance modes described earlier. Because circulation around an imperfect wire changes the normal modes from linear to circular it follows that the more circulation around the wire, the greater the separation between the two resonance frequencies. Changing circulation around the wire results in changing $d\omega$. A reasonable way to record circulation values would be to sit at some frequency where $d\omega$ changes the most then solve for circulation. This method could easily acquire circulation values at a much faster rate giving us the ability to study higher harmonic kelvin wave oscillations.

The procedure for this new technique would consist of first detecting a stable vortex along half of the wire pinned to the cell wall. A large current would then be sent through the wire to excite the high order kelvin wave oscillations along the free vortex. Then the wire would be driven by a small amplitude sine wave to detect the behavior of the kelvin waves. The amplitude of the sine wave needs to be small to avoid continually driving the kelvin wave oscillations. Before progress can be made on exploring the behavior of the kelvin waves themselves, a confirmation that this continuous mode technique produces results in line with what we expect needs to be established.

D. Frequency Sweeping

To verify the continuous technique, a LabVIEW frequency sweeping program was designed to communicate to hardware using the standard GPIB protocol. A signal is first sent to a function generator which produces a sine wave at a constant frequency. A lock-in amplifier, which reads the emf produced by the vibrating superconducting wire, is then queried for the desired amount of points. At a single frequency the voltage from the lock-in amplifier should be constant while there is no circulation around the wire. The program then tells the function generator to produce a sine wave at another frequency that is a given step size away from the first frequency. The process then continues until a sweep is completed from a start frequency to a stop frequency. The program can sweep in both direction and displays graphs recording the temperature, X component, Y Component, and amplitude of the voltage from the lock-in amplifier. A 100 to 1 voltage divider was eventually added into the circuit after the function generator, but before the superconducting wire so as to knock down the amplitude and guarantee that the wire is not driving oscillations on the vortex.

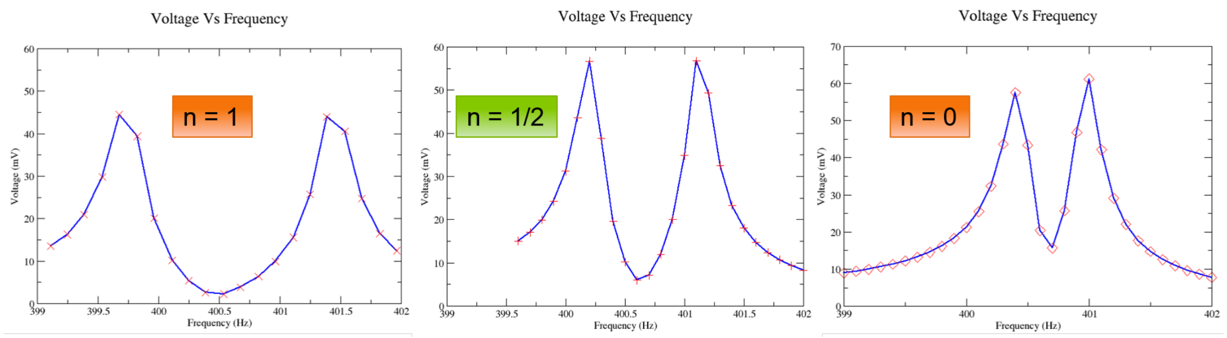

FIG. 6. A comparison between frequency sweeps at different levels of circulation with n indicating how much of the wire is covered by vortex. Each of these levels of circulation remained constant during the sweep. The double lorentzian shape was measured as expected and the more circulation the higher $d\omega$. This leads to the conclusion that changes in circulation would be detectable and at a much faster rate than the pulsed mode.

III. RESULTS AND DISCUSSION

The results of frequency sweeping at fixed amounts of circulation can be seen in Figure 6. The n value is how much of the wire is covered by a vortex. In each of these cases the circulation was not changing around the wire, but remained at a constant value allowing for a stable sweep. It is clear that the double lorentzian is there. The $d\omega$ parameter also increases with increasing amount of circulation around the wire as expected. It is also important to note that the center of the double lorentzian, ω_o, is at the same value around 400.5 Hz for each of scenarios. This parameter is most likely a property of a given wire and would change from wire to wire.

It is possible to have more than an $n = 1$ vortex. For our system, these vortices were relatively unstable and hard to attain. However, one sweep was accomplished at an $n = 2$ level and can be seen in Figure 7. The center still appears to be around 400.5 Hz, but the minimum looks shifted to below 400 Hz. This could be an artifact of the system being unstable at such high n or may be hinting at relevant physics. More sweeps at this level would necessary as reproducibility has not been established.

Figure 6 is confirmation that the continuous technique could be used and produced the expected response. However, further testing was needed to confirm that the continuous technique could see vortex phenomena. The free vortex precessing around the cell wall had been previously well studied using the pulsed mode technique. If the precession could also be seen by the continuous mode, this would provide a nice benchmark for progress on seeing kelvin wave oscillations which are much smaller compared to the precession of the free vortex around the cell. The quantity we see oscillate in both cases would be the circulation. The superconducting wire is somewhat off center in the cell so, the free vortex has to stretch to remain on the cell wall when precessing around it. This changes the amount of circulation around the superconducting wire, and produces the oscillations. In the system used this summer, the cell had differing diameters from the top half of the cell to the bottom half of the cell. The period of oscillations on both sides of the cell are pretty well understood and clearly distinguishable. Figure 8 shows a graph of a precession which then catches at a stable $n = 1$ vortex. The period of oscillations are consistent with the short side of the cell

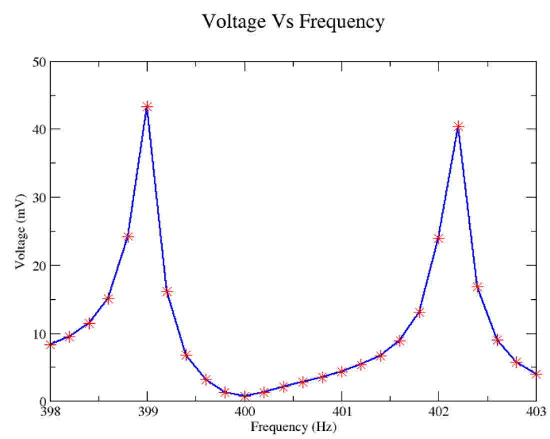

FIG. 7. A frequency sweep at $n = 2$ that shows an increase in $d\omega$ from the $n = 1$ case. The double lorentzian also appears to be centered at the same spots as the other, but has an unsymmetric minimum that may be an artifact of the instability of $n = 2$ vortices.

The next step is to devise a method that allows for a large amplitude pulse to be sent in to excite the kelvin waves, then to monitor using the continuous mode. Because a voltage divider is required to knock down the amplitude for the continuous mode, a method to send large pulses and continue to have the voltage divider in place needs to be devised. A second function generator was set up to manually allow for a pulse and have the continuous mode running the entire time where the voltages from both function generators add before the superconducting wire, but after the voltage divider. A more careful examination of how the voltages are adding

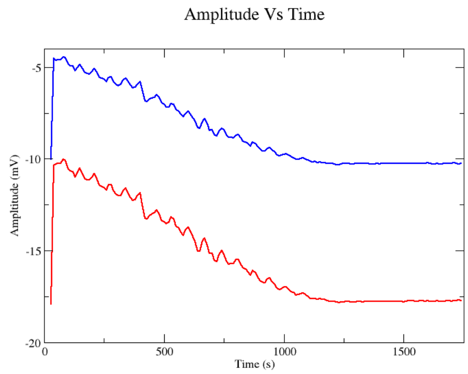

FIG. 8. A continuous mode measurement of a well-studied precession. The blue line is the X component of the amplitude and the red is the Y. The vortex clearly precesses into a stable vortex which was determined to be an $n = 1$. Detection of precession of a vortex around a cell wall using the continuous method suggests that the detection of kelvin wave oscillations is possible using this technique.

is necessary. Also, code should be written to incorporate the second function generator into the LabVIEW code for the continuous mode so as to streamline the entire process and allow for overnight runs.

IV. CONCLUSION

Switching from a pulsed mode to a continuous one by measuring in fourier space has allowed for the acquisition of circulation values at a much faster rate than before. At varying levels of constant circulation, frequency sweeps were performed and measured the expected double lorentzian response. The continuous method was also used to measure previously well studied phenomena to gauge its effectiveness. A frequency still has to be chosen to sit at that will provide the largest change in $d\omega$ which allows for the best resolution of the kelvin waves. A method for generating the kelvin waves then measuring them also still needs to be developed and tested. A larger voltage divider may be necessary to further decrease the voltage and guarantee that the continuous mode is not driving the oscillations along the free vortex.

ACKNOWLEDGMENTS

I would like to thank Dr. Rena Zieve for her encouragement and help this summer, I learned a tremendous amount and not just about physics.

I would also like to acknowledge the NSF for funding the REU program under grant PHY-1263201

V. REFERENCES

[1] Donev, Luke, *Experimental Methods and Results on the Study of Superfluid Helium*, Senior Thesis.

[2] S.C. Whitmore and W. Zimmermann, Jr, Observation of Quantized Circulation in Superfluid Helium, *Phys. Rev.* **166**,181 (1968).

[3] Neumann, Ingrid, *Interactions between a superfluid vortex and its bounding surface.*, PhD Thesis, UC Davis, 2012.

[4] I.H. Neumann and R.J. Zieve, Vortex pinning by surface geometry in superfluid helium, *Phys. Rev. B* **89**, 104521 (March 2014)