# Segregation of Binary Granular Mixtures and the Affects on Stability 

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#### Abstract

We measured how the complete segregation of two different granular materials affected the overall stability of a two-dimensional granular mixture. The segregation was implemented in the initial stages of the mixing, with larger grains aligning the bottom portion of the mixer and the smaller grains on the top portion. Type 4 segregation types were found most probable at an angle of $43^{\circ}$.


## The Introduction

Granular materials are conglomerations of discrete solid macroscopic particles that act as an indiscrete mass when interacting with other materials. Particularly, when granular materials are placed in a pile, say sand dunes, the pile is seemingly solid unless it is perturbed, say one starts walking up the sand dune, causing a sudden liquid-like flow behavior called an avalanche. This bizarre solid-liquid "phase change" is the focus of study in many granular material areas for its implications in industry and nature, such as cement mixing and studying snow avalanches.

Though there has been research within the field related to how granular materials pack together, there has not been sufficient research in how granular materials avalanche and what causes the highest probability of avalanching. In our research, we looked at simple avalanches using ball bearings in order to understand how packing of particles relate to avalanche strength to test what situations lead to the strongest piles before avalanches. ${ }^{1}$

## The Process

To analyze this concept, we focused on a two-dimensional model of avalanches using $1 / 8$ '"-diameter ball bearings, half of which were welded into double and hexagon shapes (see Figure 1) ${ }^{2}$.


Figure 1: Left- Double. Right - Hexagons.
The ball bearings were housed in a rotating drum consisting of a $1 / 8$ "-thick aluminum sheet sandwiched between two $1 / 2 "$-thick sheets of Plexiglas. A 13.96 "- diameter circle was cut out of the aluminum sheet and the ball bearings were placed in the circle. The drum rotated about the center with a rotation speed of around $65 \mu \mathrm{hz}$, or one full rotation around half an hour.

[^0]This rotating drum was positioned in front of a Logitech webcam which fed a live stream of photographs into a computer. These images were analyzed using a Python program, granmatter_python.py. First, the program roughly estimated the angle size of the material pile, storing the information for future comparison. When there is an avalanche, the angle size decreases, triggering the program to analyze the past 100 photos in its cache in order to pinpoint the beginning and end of the avalanche. When the avalanche point is found, photos, angle size, time, date, and index number of the pile before and after the avalanche is stored into a separate file. ${ }^{3}$

## The Avalanches

For this experiment, the ball bearings were initially configured to be completely segregated: hexagonshaped bearings were lined near the circumference of the circle whereas the double-shaped bearings were lined near the center of the circle (Figure 2.a). As the tumbler rotates, five other configurations are formed, called type 1, type 2, type 3, type 4, semi-steady, and steady states (Figure 2.a-2.f).


[^1]

Figure 2: a. Type 1, b. Type 2, c. Type 3, d. Type 4, e. Type 5/Semi-Steady State, f. Type 6/Steady State
Types 1 through 4 states are dubbed distinct segregated states. Type 1 states, the initial state, has two parallel layers of materials with a clear divide between hexagons and doubles. Type 2 states have doubles mostly conglomerated below the hexagons. Type 3 states are supported by hexagons, with doubles in the center, and hexagons on top. Type 4 states are supported by hexagons much like type 2 states are supported by doubles.

The semi-steady state is a configuration that happens after around 50 avalanches. These states lie between type 1 through 4 configurations and the steady state configuration. Particularly, characteristics of type 1 through 4 configurations (how the pile is supported) can be seen but the materials are beginning to integrate to form a steady state.

The steady state configuration is markedly noted as the configuration of hexagons lining the outer circumference of the circle, enclosing doubles towards the center. After many hours of rotation, completely segregated materials will mix to form this particular configuration. Once this state is reached, no other state will be formed, which is why the name "steady state" is given to this configuration: this state is the most "natural" and likely state of the ball bearings.

As particle packing has been thoroughly researched, the key goal of this experiment was to find the relationship between how particles pack, specifically the configuration of the granular materials, and how this related to angle size. To measure this, images in each run were sorted per angle size and then categorized by different segregation types. Then, each segregation types were compared to the angle size to find a relationship between these two categories. Each image was inspected by eye.

## The Analysis

After each run, each image was classified by before-angles ${ }^{4}$ sizes and each set of similar angles were analyzed to filter through the different types that were present in each angle size set. Four runs were analyzed, with angles ranging from $34^{\circ}$ to $59^{\circ}$. Figure 8 shows the number of segregation types 1 through 6 per angle size.

[^2]

Figure 3: a. First Run, b. Second Run, c. Third Run, d. Fourth Run
As one notices from figure 3, the most probable segregation type present in each run is type 6 , the steady state, which is obvious because the steady state formation is the most natural state of a hexagon-double pile. Semi-steady state types are the second most probable types during a run. However, what we are most interested in for this paper is the probability of the distinct segregated states in each run. To look at these four types more obviously, figure 4 shows the number of distinct segregation states per angle size.



Figure 4: a. First Run, b. Second Run, c. Third Run, d. Fourth Run
Judging from Figure 9.a to 9.d, for each different run, it seems that type 4 segregation states are the most probable in general and type 2 segregation states are the second most probable in general. However, there is not an obvious relationship between distinct segregation states and angle size. To find this relationship more obviously, the data from all four runs was summed together and plotted against angle size to give Figure 5.

Total Number of PreAvalanche Piles per Angle Size Without Semi-Steady and Steady S


Figure 5: Total number of Avalanche Pile per Angle Size
What one notices from Figure 5 is that the most probable segregation types are type 4 's at $43^{\circ}$. There is an interesting peak of type 2 's at $45^{\circ}$, which may have been a result of categorization error. Interesting enough, when comparing figure 5 to figure 6.b, the most probable segregation type is also related to the most probable angle the summation of runs will have. If comparing figure 4 and figure 6 .a however, that relationship is not present.

Initially, the question that drove this experiment was to find a relationship between the different segregation types and avalanche angles. However, in order to find a relationship between the two phenomenons, we would need to have a larger sample size of avalanches. So instead, the analysis focused
more on the statistics of the groups rather than finding a reason for correlations between segregation types and avalanche angle sizes.


Figure 6: a. Most Probable Angle Size per Run, b. Most Probable Angle Size in Total

## The Conclusion

After analyzing four different runs and a summation of the four runs, it seems that the most probable angle in a summation of four runs is $43^{\circ}$. The most probable distinct segregation type, type 4 , is also most probable at $43^{\circ}$. Overall, steady-state and semi-steady state segregation types are by far the most probable types that will occur during a run. However, when comparing distinct segregation states, it seems that type 4 and 2 segregations will be the most probable.

A reason why type 4 and 2 are most probable may be because, to the human eye, they are more discernible than type 1 and 3 states. Many images that could have been type 1 and 3 states were ultimately characterized as semi-steady states due to the lack of differentiation between piles. In addition, though a human sorter is consciously trying to be consistent, inconsistencies can occur due to the volume of images being sorted through. For future projects, this human error can be resolved if the entire process becomes computerized, so that an objective consistency be maintained throughout the analysis process.

## Appendix

Sample image data

| Avalanche <br> $\#$ | \# of <br> Errors | Before <br> Avalanche <br> Index \# | Before <br> Avalanche <br> Angle Size | After <br> Avalanche <br> Index \# | After <br> Avalanche <br> Angle Size | Time and Date |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 75 | 43.45967 | 113 | 30.58891 | $1918: 39: 122013$ |
| 2 | 0 | 82 | 45.14652 | 143 | 33.146 | $1918: 41: 282013$ |
| 3 | 0 | 76 | 42.47972 | 128 | 34.98906 | $1918: 43: 052013$ |
| 4 | 0 | 77 | 47.10556 | 111 | 31.11049 | $1918: 44: 592013$ |
| 5 | 0 | 69 | 47.34748 | 98 | 34.13013 | $1918: 47: 312013$ |
| 6 | 0 | 82 | 44.79325 | 117 | 33.97008 | $1918: 49: 102013$ |
| 7 | 0 | 74 | 50.54498 | 103 | 32.71846 | $1918: 51: 472013$ |
| 8 | 0 | 81 | 54.68105 | 112 | 28.876 | $1918: 55: 102013$ |
| 9 | 0 | 74 | 41.8057 | 97 | 32.21563 | $1918: 57: 062013$ |
| 10 | 0 | 78 | 47.82391 | 114 | 30.42373 | $1918: 59: 242013$ |

Sample Data for Number of Images per Angle Size Graphs

| Before Angle Size | Number of Images |
| ---: | ---: |
| 35 | 1 |
| 37 | 5 |
| 38 | 11 |
| 39 | 32 |
| 40 | 50 |
| 41 | 59 |
| 42 | 77 |
| 43 | 84 |
| 44 | 80 |
| 45 | 78 |
| 46 | 49 |
| 47 | 41 |
| 48 | 33 |
| 49 | 24 |
| 50 | 12 |
| 51 | 16 |
| 52 | 3 |
| 54 | 3 |
| 55 | 2 |
| 59 | 1 |
| 35 | 1 |

Sample Data for Number of Types per Angle Size Graphs

| Angle | Type1 | Type2 | Type3 | Type4 | Semi-Steady |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 35 | 0 | 0 | 0 | 0 | 0 |
| 37 | 2 | 0 | 0 | 1 | 1 |
| 38 | 0 | 0 | 0 | 2 | 1 |
| 39 | 0 | 1 | 1 | 2 | 4 |
| 40 | 1 | 3 | 2 | 4 | 8 |
| 41 | 1 | 1 | 0 | 5 | 9 |
| 42 | 3 | 4 | 4 | 5 | 6 |
| 43 | 2 | 2 | 3 | 5 | 22 |
| 44 | 1 | 2 | 6 | 5 | 9 |
| 45 | 2 | 6 | 2 | 5 | 13 |
| 46 | 1 | 2 | 0 | 2 | 11 |
| 47 | 0 | 3 | 2 | 3 | 9 |
| 48 | 0 | 3 | 2 | 2 | 10 |
| 49 | 0 | 1 | 1 | 2 | 4 |

Semi-Steady State vs. Type 1 Image


Semi-Steady State vs. Type 3 Image


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[^0]:    ${ }^{1}$ General information references in Reference section.
    ${ }^{2}$ We used hexagons and doubles because singles have very bad packing densities and, when hexagons were packed with other configurations of the ball bearings, the packing density did not change.

[^1]:    ${ }^{3}$ Or more information about the program, please read Victoria Winbow's and Bryce Kalmbach's papers. Links in the Reference section.

[^2]:    ${ }^{4}$ Decimal places were rounded to the nearest whole number. If an angle read $35.462^{\circ}$, it was rounded to $35^{\circ}$. If the angle was $35.538^{\circ}$, it was rounded to $36^{\circ}$.

