Dark Sector Coupling

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Abstract

An interaction between dark matter and dark energy is introduced and its impact on cosmic microwave background anisotropy is considered. Using a modified version of the CLASS Boltzmann integrator, we reproduce the results of a previous study. We discuss further modifications that must be made to CLASS and prepare to perform a fit to Planck data for each of the eight forms of the coupling that are discussed.

I. Introduction

Dark matter and dark energy are the dominant constituents of the universe, making up about 95% of its mass. However, we know little about dark matter and dark energy.

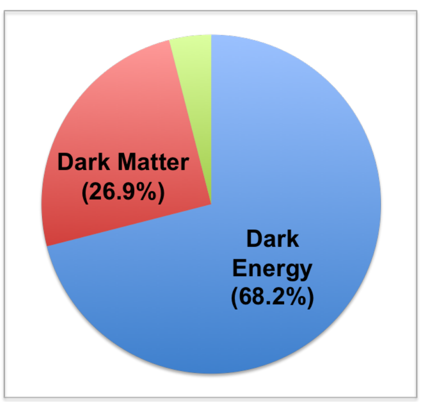

Figure 1: The Contents of the Universe: *Dark energy (blue) and dark matter (red) are the dominant constituents. Together they comprise 95.1% of the universe. The remaining 4.9% (shown in green) includes baryonic matter and radiation.*

Dark Matter Although we do not know what dark energy is made up of, we know that it exists because we can measure its gravitational effects. Dark matter was first proposed to explain the following conundrum: We can calculate the mass of galaxies in a galaxy cluster by measuring their orbital velocities, and we can calculate the amount of visible matter in these galaxies based on the amount of light they emit. Comparing these two values, we find that the mass of visible matter in these galaxies falls significantly short of the total mass. Dark matter, so-called because it does not interact via the electromagnetic force, accounts for this missing mass.

Dark Energy The source of dark energy is also a mystery. One proposed source of dark energy will be discussed in Section III. We can see the effects of dark energy in the expansion rate of the universe: when we measure the velocities of distant supernovae, we can see that the expansion of the universe is accelerating (the 2011 Nobel Prize in Physics was awarded for this discovery). Dark energy must be driving the acceleration of the universe's expansion.

In the standard model of cosmology, dark matter and dark energy do not interact with ordinary matter, except via gravity. In the standard model, dark matter and dark energy also don't interact with each other. In Section IV, I will introduce a coupling between dark energy and dark matter, and will explore the impact of this coupling on the evolution of the dark energy and dark matter densities.

II. Linear perturbation theory

On very large scales (greater than 10 megaparsecs), the universe appears homogeneous: its contents and structure do not vary much throughout space. Therefore, we will take a homogenous universe as a zero-order approximation for the universe. In this zero-order universe, the densities of each species are constant throughout space, e.g.

$$\frac{\partial \rho_{DM}}{\partial x^i} = 0\ ;\ \frac{\partial \rho_{DE}}{\partial x^i} = 0 \quad \text{for } i \in \{1,2,3\} \tag{1}$$

We can describe the expansion of the universe using the scale factor $a(t)$ (see Figure 2). The Hubble parameter H, named for Edwin Hubble, who first observed that the universe is expanding, also describes the expansion rate. H depends on the density of each species (e.g. baryons, dark matter, etc.) in the universe:

$$H \equiv \frac{1}{a}\frac{da}{dt} \propto \left(\sum_i \rho_i \right)^{1/2} \tag{2}$$

When we introduce a coupling between dark matter and dark energy in Section IV, the evolution of ρ_{DM} and ρ_{DE} will be affected, which will in turn impact the Hubble parameter and the evolution of a.

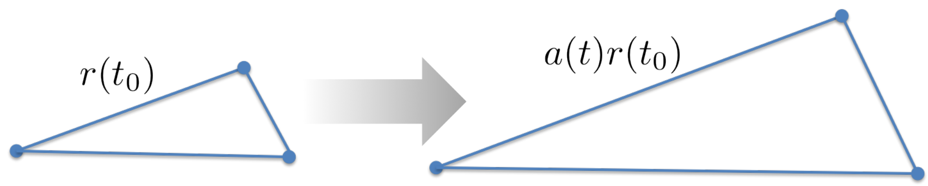

Figure 2: The expansion of the universe will cause the side of a triangle to expand from its initial length $r(t_0)$ to length $a(t)r(t_0)$ at a later time t.

However, the zero-order universe does not tell us the whole story, since the universe is only approximately homogeneous. Now we have to add perturbations to the zero-order universe. The distance scales we are considering are large enough that we only need to consider first order perturbations, and all higher order terms can be neglected.

For both dark matter and dark energy, I will split the stress-energy tensor $T^{\mu\nu}$ into a zero-order component $T^{\mu\nu}_{(0)}$ and a first order perturbation $\delta T^{\mu\nu}$:

$$\begin{aligned} T^{\mu\nu}_{DM} &= T^{\mu\nu}_{(0)DM} + \delta T^{\mu\nu}_{DM} \\ T^{\mu\nu}_{DE} &= T^{\mu\nu}_{(0)DE} + \delta T^{\mu\nu}_{DE} \end{aligned} \tag{3}$$

The T^{00} component of each of these tensor equations gives the total energy density, broken into a zero-order component ρ and a first-order perturbation $\delta\rho$:

$$\begin{aligned} \rho^{tot}_{DM} &= \rho_{DM} + \delta\rho_{DM} \\ \rho^{tot}_{DE} &= \rho_{DE} + \delta\rho_{DE} \end{aligned} \tag{4}$$

By considering the zero-order and first-order components of the stress-energy tensor separately, I will work out separate differential equations for the zero-order density ρ and for the density perturbations, which depend on $\delta\rho$.

To perform these calculations, we will also need to know the equation of state parameter w for both dark energy and dark matter. The equation of state parameter for a given species is defined as the pressure over the density:

$$w_i = p_i/\rho_i \tag{5}$$

Because dark matter has zero pressure ($p_{DM} = 0$), the equation of state parameter for dark matter is zero. The equation of state parameter for dark energy will be discussed in detail in Section III.

III. Quintessence

In the standard model of cosmology, the density of dark energy is a "cosmological constant," Λ: it does not change as the universe expands. In order to introduce coupling between dark energy and dark matter, I will discuss another model of dark energy: a scalar quintessence field, ϕ, with potential $V(\phi)$. We assume that the quintessence field is in "slow roll," so that

$$V'(\phi)/V(\phi) \ll 1\ ;\ V''(\phi)/V(\phi) \ll 1 \tag{6}$$

The equation of state parameter for the quintessence field depends on the potential:

$$w_Q = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{7}$$

Many different forms of the potential $V(\phi)$ result in a "tracker solution" for w_Q: that is, for many choices of the potential, w_Q converges to the equation of state parameter of the dominant species in the universe [1]. Therefore, it is practical to choose a tracker solution for w_Q, which is valid for many possible potentials, rather than a single form for $V(\phi)$.

To calculate the stress-energy tensor of the quintessence field, we follow the discussion of inflation in [2]:

$$T_Q^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}\frac{\partial\phi}{\partial x^\alpha}\frac{\partial\phi}{\partial x^\beta} - \frac{1}{2}g^{\mu\nu}\left(\frac{\partial\phi}{\partial x^\alpha}\frac{\partial\phi}{\partial x^\beta}g^{\alpha\beta} + 2V(\phi)\right) \tag{8}$$

We choose $g_{\mu\nu}$ to be the perturbed Friedmann-Robertson-Walker metric to first order:

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)(dx^2+dy^2+dz^2) \tag{9}$$

including first order metric perturbations Ψ and Φ.

Before introducing a coupling between dark matter and dark energy, we consider the case of no coupling. For the case of no coupling, we assume that neither dark matter nor the quintessence field interact with matter or radiation, and that there are no interactions between dark matter and the quintessence field. Under these assumptions, we have

$$\begin{aligned} \nabla_\mu T_{DM}^{\mu\nu} &= 0 \\ \nabla_\mu T_{DE}^{\mu\nu} &= 0 \end{aligned} \tag{10}$$

where we use $T_{DE}^{\mu\nu} = T_Q^{\mu\nu}$, calculated in (8). $T_{DM}^{\mu\nu}$ can be calculated using the familiar formula for the stress-energy tensor:

$$T^{\mu\nu} = pg^{\mu\nu} + (\rho+p)u^\mu u^\nu \tag{11}$$

where p is the pressure ($p = 0$ for dark matter), ρ is the dark matter density, and u^ν is the dark matter four velocity.

Again following [2], we derive the equations of motion for the zero- and first-order evolution of the densities by taking the $\nu = 0$ component of (10) and breaking up $T^{\mu\nu}$ as in (3).

Zero-order We find the time evolution of the zero-order density of dark matter (ρ_{DM}) and of dark energy ($\rho_{DE} = \rho_Q$) by considering only the zero-order terms in (10). This gives

$$\begin{aligned} \dot{\rho}_{DM} + 3aH\rho_{DM} &= 0 \\ \dot{\rho}_{DE} + 3aH(1+w)\rho_{DE} &= 0 \end{aligned} \tag{12}$$

where $w = w_Q$ is the equation of state parameter of the quintessence field. The equation of state parameter is zero for dark matter, since $p_{DM} = 0$.

First-order The density perturbations are described the first-order parameters δ and θ. The density perturbation δ is defined as

$$\delta = \delta\rho/\rho \tag{13}$$

The velocity gradient θ is defined as

$$\theta = -ik_j v^j \tag{14}$$

where v_j is the spatial component of the four velocity of dark matter (for θ_{DE}), or $v^j = ik^j\delta\phi/\dot{\phi}$ with ϕ the quintessence field (for θ_{DM}). Considering only the first-order terms in (10) yields differential equations for δ_{DM} and δ_{DE}:

$$\begin{aligned} \dot{\delta}_{DM} &= -(\theta_{DM} - 3\dot{\Phi}) \\ \dot{\delta}_{DE} &= -(1+w)(\theta_{DE} - 3\dot{\Phi}) - 3aH(1-w)[\delta_{DE} + aH(3(1+w))\tfrac{\theta_{DE}}{k^2}] \end{aligned} \tag{15}$$

We can find the differential equation for θ_{DM} by following chapter 4 of [2], and for θ_{DE} by taking the time derivative of (14):

$$\begin{aligned} \dot{\theta}_{DM} &= -aH\theta_{DM} + k^2\Psi \\ \dot{\theta}_{DE} &= 2aH\theta_{DE} + \tfrac{k^2\delta_{DE}}{1+w} + k^2\Psi \end{aligned} \tag{16}$$

IV. Introducing the coupling

When we introduce a coupling between dark matter and dark energy, we no longer have $T^{\mu\nu}_{DM}$ and $T^{\mu\nu}_{DE}$ separately conserved as in (10), since energy is transferred from dark matter to dark energy or vice versa. Instead, we have

$$\nabla T^{\mu\nu}_{DM} + \nabla T^{\mu\nu}_{DE} = 0 \tag{17}$$

which can be broken down into

$$\begin{aligned} \nabla_\mu T^{\mu\nu}_{DM} &= Q^\nu \\ \nabla_\mu T^{\mu\nu}_{DE} &= -Q^\nu \end{aligned} \tag{18}$$

where Q^ν is not necessarily zero.

We consider coupling of the form

$$Q^\nu = Qu^\nu/a \tag{19}$$

where u^ν is the four velocity of either the dark matter or dark energy fluid. Dividing by a cancels the time dependence of u^ν. These appear to be the only natural choices for the four-vector Q^ν, since u^ν_{DE} and u^ν_{DM} are the only four-vectors that are relevant to dark matter/ dark energy coupling.

It is clear from (18) that the scalar Q in (19) must include units of energy density. We therefore choose Q to be proportional to either the dark energy density or the dark matter density:

$$Q \propto \rho_{DE}\ ;\ Q \propto \rho_{DM} \tag{20}$$

We consider two choices for the proportionality of Q to ρ, where ρ is the density of either dark matter or dark energy. First, we take Q to be

$$Q = \xi a H \rho \tag{21}$$

where a is the scale factor, H the Hubble parameter, and ξ some scalar. This is the form of Q discussed in [3] and [4]. Although this form for Q is more common in the literature, there is no physical justification for why the coupling might depend on global parameters like a and H. The second choice for Q does not encounter this problem. In this case, the dark energy/ dark matter coupling is modeled as a particle interaction, with interaction rate Γ:

$$Q = \Gamma\rho \tag{22}$$

However, calculating the initial conditions for the density perturbations δ and θ becomes significantly more complicated if we choose the second form for Q. All eight different forms of Q^ν that we have discussed are summarized in Figure 3.

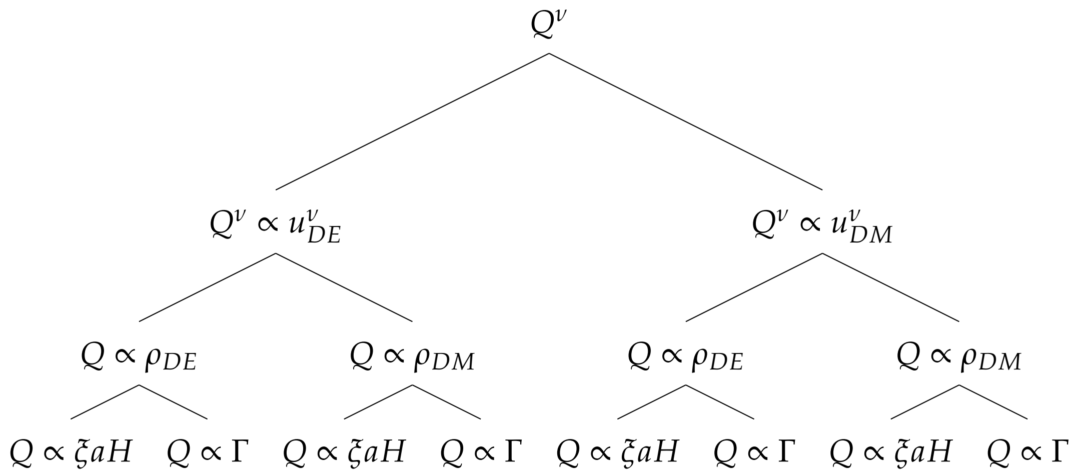

Figure 3: This tree shows each of the choices for Q^ν that we consider.

Zero-order When we include dark matter/ dark energy coupling in any of these forms, the equations describing the background evolution of ρ_{DM} and ρ_{DE} in (12) each gain an additional term:

$$\begin{aligned} \dot{\rho}_{DM} + 3aH\rho_{DM} &= Q \\ \dot{\rho}_{DE} + 3aH(1+w)\rho_{DE} &= -Q \end{aligned} \tag{23}$$

For no coupling, $Q = 0$ and these equations simplify to (12).

First-order The equations for the perturbations δ and θ also gain additional terms when we include dark matter/ dark energy coupling:

$$\begin{aligned} \dot{\delta}_{DM} &= -(\theta_{DM} - 3\dot{\Phi}) + \tfrac{Q}{\rho_{DM}}(\tfrac{\delta Q}{Q} - \delta_{DM} + \Psi) \\ \dot{\delta}_{DE} &= -(1+w)(\theta_{DE} - 3\dot{\Phi}) - \tfrac{Q}{\rho_{DE}}(\tfrac{\delta Q}{Q} - \delta_{DE} + \Psi) \\ &\quad -3aH(1-w)[\delta_{DE} + aH(3(1+w) + \tfrac{Q}{\rho_{DE}})\tfrac{\theta_{DE}}{k^2}] \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{\theta}_{DM} &= -aH\theta_{DM} + (1-b)\tfrac{Q}{\rho_{DM}}(\theta_{DE} - \theta_{DM}) + k^2\Psi \\ \dot{\theta}_{DE} &= aH(2 + \tfrac{(1+b)Q}{(1+w)aH\rho_{DE}})\theta_{DE} + \tfrac{k^2\delta_{DE}}{1+w} + k^2\Psi - \tfrac{bQ\theta_{DM}}{\rho_{DE}(1+w)} \end{aligned} \tag{25}$$

For no coupling, $Q = 0$ and $\delta Q = 0$, so (24) and (25) equations simplify to (15) and (16) respectively. In equation (25), $b = 0$ for $Q^\nu \propto u^\nu_{DE}$ and $b = 1$ for $Q^\nu \propto u^\nu_{DM}$, as in [3].

V. CLASS and the CMB

The CLASS ("Cosmic Linear Anisotropy Solving System") Boltzmann integrator [5] computes cosmological observables by integrating differential equations including (12), (15), (16), and many others. We are interested in one observable in particular: the Cosmic Microwave Background (CMB) temperature power spectrum.

Cosmic Microwave Background The early universe was composed of a hot plasma containing free electrons, protons, and photons. Around $z = 1100$, the universe had cooled sufficiently to allow free electrons and protons to combine to form neutrons, Hydrogen, and Helium. Without free electrons to scatter with, photons began to free stream: since that time, these photons have travelled to us virtually unimpeded. These photons are known as cosmic microwave background (CMB) photons. The CMB temperature power spectrum is a very useful observable for our purposes because the temperature fluctuations in the CMB closely trace the perturbations that we are interested in.

In order to incorporate dark matter/ dark energy coupling into CLASS, we modified the equations of motion for ρ_{DM} and ρ_{DE} to include the additional terms gained in (23). Similarly, we modified the equations of motion for δ_{DM}, δ_{DE}, θ_{DM}, and θ_{DE} to include the additional terms gained in (24) and (25). We can now choose to run CLASS with any of the eight forms of Q^{ν} shown in Figure 3 for a specified value of ξ or Γ. This study will be continued in fall 2013, and we will use this version of CLASS to find the best fit values for each model, and the corresponding CMB spectra. The details of these next steps are discussed in Section VI.

As a test of the code, we use coupling of the form $Q^{\nu} = \xi a H \rho_{DE} u^{\nu}_{DM}$ and the best fit parameter values from [4], and compare the resulting spectrum to the Planck best-fit spectrum (Figure 4). The evolution of ρ_{DM} and ρ_{DE} in each model is compared in Figure 5.

VI. Conclusions and Further Steps

Our modified version of CLASS produces reasonable spectra for scenarios that do not include massive neutrinos. However, when we include massive neutrinos and compare the output of our code to spectra computed using CAMB (another commonly used Boltzmann integrator, used in [4]), comparisons fail due to inherent differences between CLASS and CAMB. Before proceeding with the study, we need to understand the differences in the treatment of massive neutrinos in CLASS and CAMB.

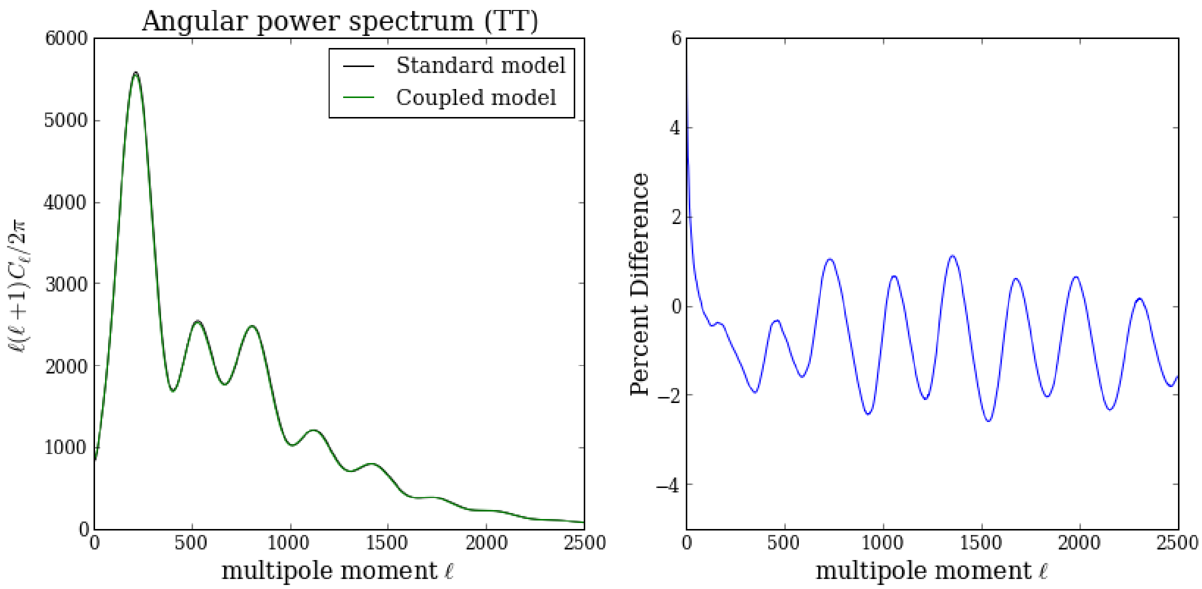

Figure 4: (Left) We overlay the CMB temperature power spectra of the standard model (Planck best-fit spectrum) and a coupled model, which uses $Q^\nu = \xi a H \rho_{DE} u^\nu_{DM}$ and the best fit parameters from [4]. (Right) The percent difference (as a function of multipole) between the two spectra shown on the left.

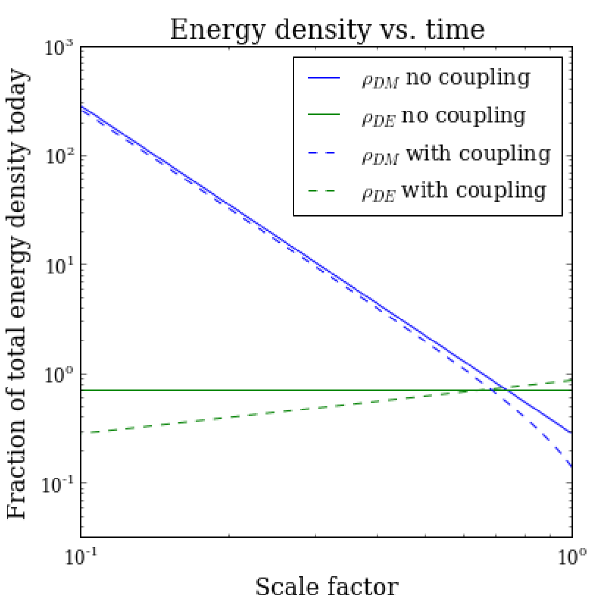

Figure 5: The density of dark matter and dark energy as a function of the scale factor for two models: the dashed lines show the densities for the Planck best-fit model, and the solid lines show the densities for a model using $Q^\nu = \xi a H \rho_{DE} u^\nu_{DM}$ and the best fit parameters from [4]. The density of dark matter is diluted when we include the coupling, since we have energy transferred from dark matter to dark energy.

CLASS also needs to be further adjusted to run with a tracker solution for w (it is currently set up to run only for w linear in a).

The next step will be to perform a joint probability distribution for the cosmological parameters for each of our eight models. Interesting models, i.e. models favoring Q significantly different from zero, will point towards the existence of dark matter/ dark energy coupling, and will merit further investigation. The existence of dark matter/ dark energy coupling would have exciting implications for cosmology, particle physics, and beyond.

References

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