The Effects of Cell Perturbations on Superfluid Helium Vortices

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When forced to circulate in a cell, a superfluid vortex may behave differently depending on the cell structure. To measure these effects on the ability of a vortex to pin, various perturbations were added to a series of cells and the vortex's change in pinning behavior was measured. However, there was insufficient time to build a usable cell and collect data.

I. INTRODUCTION

Helium is unique among the elements in that it does not freeze at low temperatures and pressures, remaining a liquid all the way down to absolute zero at standard pressure. It also has the lowest boiling point, 4.22 K[1], and is the only element to has a superfluid phase transition (2.17 K). While a solid phase does exist, it requires extreme pressure, around 25 atmospheres. A phase diagram for helium is shown in figure 1.

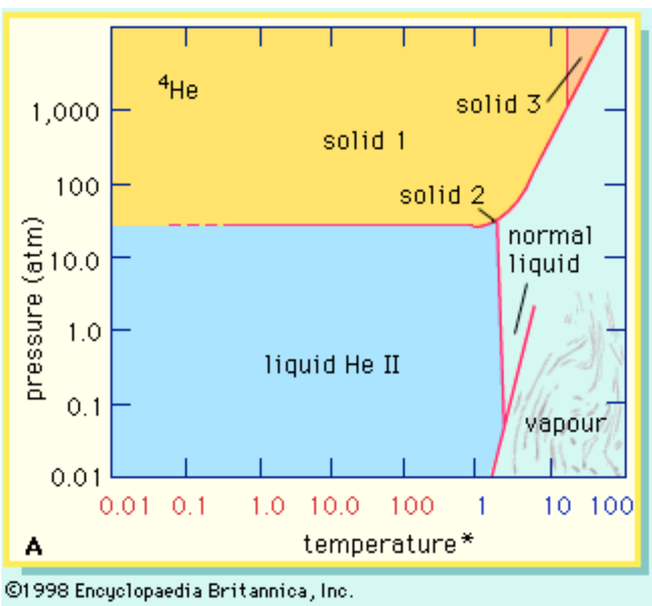

FIG. 1: The phase diagram of helium[2] shown on a log-log plot. *Temperature is measured in Kelvin above 1K, and in mK below 1K.

Superfluids have a number of exotic properties, such as zero viscosity, the ability to climb walls, incredible heat capacity, and quantized circulation[3]. It is the quantized circulation that makes superfluid helium such a desirable substance for research on vortices.

Vortices, which are merely circulating velocity vectors, arise in a surprisingly large number of dynamic systems, from air and liquid flow to magnetic field lines. However, vortices are poorly understood from an analytical standpoint. The Naiver-Stokes equations, which govern fluid flow in general, are very opaque and difficult to solve for anything beyond the most trivial cases[3]. In addition, they are notoriously hard to numerically integrate, which makes modeling vortices difficult computationally.

By using a superfluid, the quantized circulation limits the possible behavior of any vortices. Thus we can study and observe vortex behavior in a relatively simplified system.

II. QUANTIZED CIRCULATION

Circulation is defined as

$$K \equiv \oint \vec{v} \cdot \vec{dl}. \tag{1}$$

Let us begin with the simplest case of circulation. Assume there is a vortex core at the center of some fluid, the fluid is moving uniformly around the core, and there are no boundary effects, as seen in figure 2.

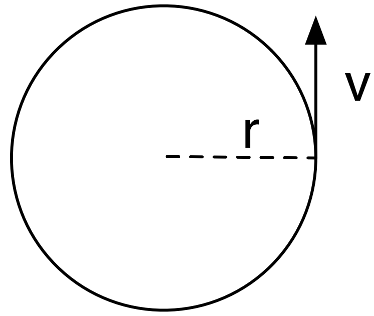

FIG. 2: A diagram of the circulation in a fluid. The fluid is moving at a tangential velocity $v(r)$ a distance r from the vortex core. Boundary effects are being ignored for simplicity.

Since the velocity vector is tangential to the shown circle, the dot product of $\vec{v}(r) \cdot \vec{dl}$ reduces to the scalar multiplication of $v(r)\ dl$. Additionally, the magnitude of $\vec{v}(r)$ must be constant everywhere on the circle, by symmetry. Thus equation 1 simplifies to

$$K = v(r) \oint dl. \tag{2}$$

The line integral $\oint dl$ now becomes trivial, as it is merely the perimeter of the circle, further simplifying equation 2 to

$$K = 2\pi r \, v(r). \tag{3}$$

Though a different derivation not shown here, one can prove that by solving the quantum wave form, the circulation of of superfluid helium is quantized[3], specificity that

$$K = \frac{nh}{M_{he}}, \tag{4}$$

here n is the quantum number (an integer), h is Planck's constant, and M_{he} is the mass of helium. Combining this quantization condition with equation 3 and solving for $v(r)$ yields

$$v(r) = \frac{nh}{2\pi r M_{he}}. \tag{5}$$

Thus, for any given radius, the velocity of the superfluid is constrained to integer multiples of the above quantity. From equation 5, it follows that

$$v(r) \propto \frac{1}{r}. \tag{6}$$

Therefore, the further away from the vortex core, the slower the fluid flows. This may be counterintuitive, as it differs from solid state rotation, such a spinning disk, where the tangental velocity increases linearly with the radius.

Most importantly, equation 6 implies that the majority of the kinetic energy lies near the vortex core, as that is where the fastest moving fluid is located[5]. If the vortex core were to be replaced with a stationary object, such as a wire, then there is now much less kinetic energy in the system. Thus, when vortices are forming, they are much more likely to form around stationary objects, as it allows for a lower energy state than freely floating vortices.

III. EXPERIMENTAL

A Cooling Helium

Cooling the helium down to an adequately low temperature is a lengthy process. The helium is first pumped into a charcoal cold trap, which is cooled in liquid nitrogen. This cold trap cleans the helium, as oxygen or nitrogen molecules are condensed and absorbed into the charcoal. The cleansed helium is then pumped into a small cell, with a wire stretched down the middle, similar to the one shown in figure 3. The cell is then cooled to 320 mK by a combination of submerging it in liquid helium and evaporation. Being well below the transition temperature, the helium becomes superfluid.

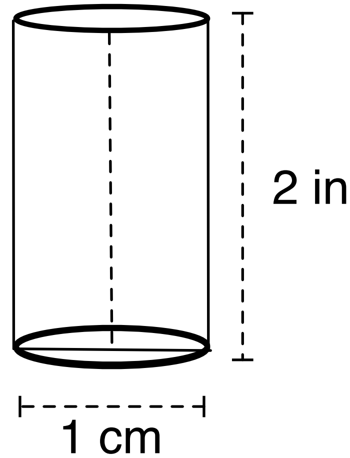

FIG. 3: A diagram of the cell used to contain the superfluid helium. A 10 micrometer diameter wire in the center stretches from the top to the bottom. There is a small hole(not shown) at the top of the cell that permits helium to enter.

B Inducing Circulation

Once the cell is adequately cold and filled with helium, it is then manually rotated to induce circulation in the superfluid. The swirling fluid forms vortices, which typically pin on the wire, as shown in figure 4. The amount of circulation on the wire almost always has a quantum number of one, though rarely a vortex with a quantum number of two is seen. Measuring circulation is done through the magnus force.

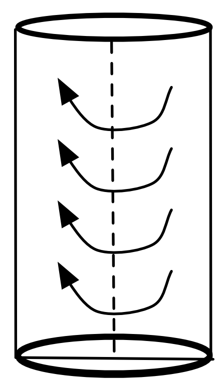

FIG. 4: A vortex circulating around the interior wire. The vortex usually has a quantum number n of one, the slowest possible vortex that can exist..

C Magnus Force

If we were to look at a cross sectional view of the cell shown in figure 4, we would see something like that shown in figure

5a. The wire is stationary while the fluid circles around it. However, if the wire to move, as in figure 5b, the fluid velocity relative to the wire is greater on one side. This velocity inequality creates a pressure difference, thus pushing the wire, as shown in 5c. The force from this pressure difference is the magnus force.

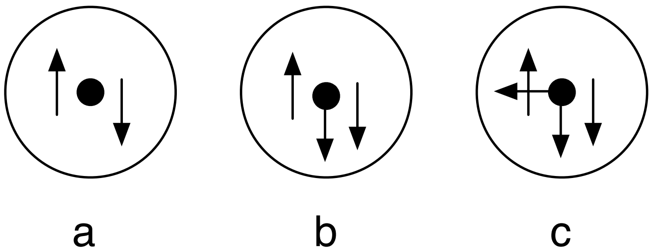

FIG. 5: A cross section view of cell shown in figure 4. In figure a, the wire is stationary, and the velocity of the fluid of the right and left sides have equal magnitude. In figure b, the wire is moving downward, thus relative to the wire, the fluid on the left side is traveling faster than the fluid on the right. This creates a pressure difference, pushing the wire to the left, as shown in figure c

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To start the wire moving, an electromagnet is placed next to the cell, and a current is sent though the wire. This creates a force on the wire, where the force is directly proportional to the cross product of the current and the electromagnet's magnetic field.

It is important to note that the magnus force is always perpendicular to the direction of motion. Therefore, if the wire is vibrating, the magnus force is perpendicular to the plane of oscillation, and the plane will rotate. Thus, a vibrating wire will precess around the cell.

D Measuring Circulation

A vibrating wire in a magnetic field generates an electromotive force[3] (or emf). Since an electromagnet is already nearby to vibrate the wire, producing the magnetic field is easy. The magnitude of the generated emf depends on the angle in which the wire vibrates relative to the magnetic field. For example, the wire in figure 6a will generate a large emf, whereas the wire in figure 6b will generate zero emf.

As was established earlier, the plane in which the wire oscillates will precess, thus the wire will move from the state in figure 6a to the state in figure 6b and back again. By measuring the voltage across the wire, and looking at the time required to move from a voltage peak to a voltage minimum, the circulation can be inferred . Some example emf data is shown in figure 7.

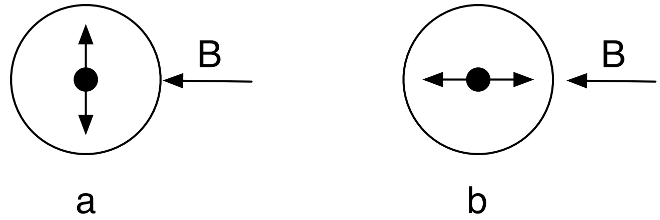

FIG. 6: The vibrating wire in figure a will generate a large emf because the plane of vibration is perpendicular to the magnetic field, whereas the wire in figure b is vibrating parallel to the field, thus it will generate no emf.

E Pinning

After inducing a vortex to form on the cell, it is possible to dislodge it from one end of the wire and pin to the wall, as shown in figure 8. The vortex can come off either end, and sometimes it will come off both, leaving just the middle attached to the wire. After, coming off the wire, the spot where the vortex terminates on the wall will precess around the cell, slowly unwinding off the wire.

Forcing a wire to dislodge can be easily done by gradually heating the cell.

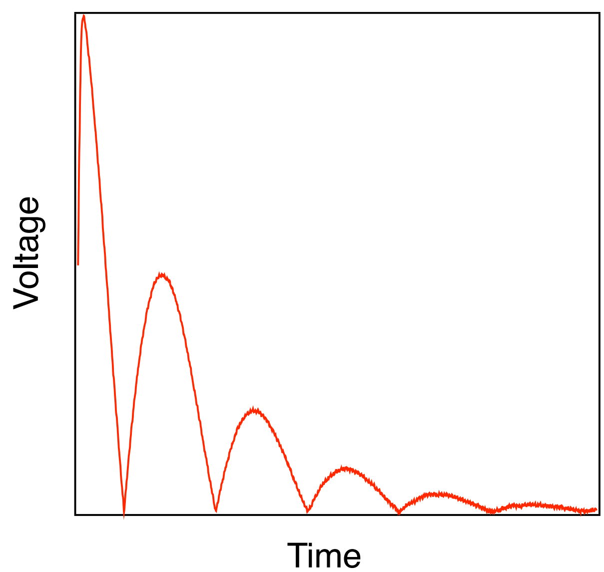

FIG. 7: A graph showing the emf measured on a precessing vibrating wire in a magnetic field. The maximums occur where the wire is perpendicular to the magnetic field, and the minimums occur when the wire is parallel to the magnetic field.The maximum decays over time because the helium is creating a drag force. Units has been omitted.

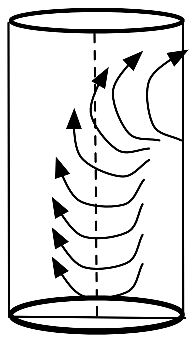

FIG. 8: A diagram of a vortex being dislodged from the top of the wire and pinning on the wall.

IV. PERTURBATIONS IN CELL DESIGN

Now that we know that the vortex can come off the wire, we would like to know how perturbations in the cell design affect the dislodging ability of the vortex. To test this, the simplistic cell design shown in figure 3 was altered by reducing the diameter for the bottom half of the cell, as shown in figure 9a. In addition, some cells had one or more of their flat ends replaced by pointed ends, as shown in figure 9b.

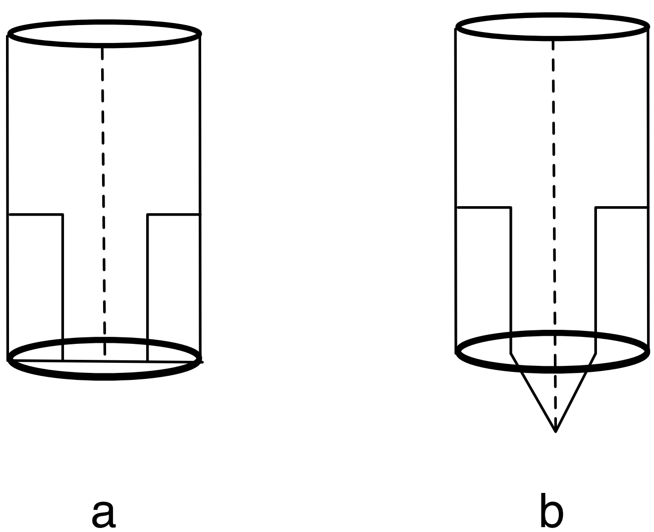

FIG. 9: Two types of cells generated to test the effects of perturbations on vortex dislodging. In cell a, the diameter is reduced at the midpoint. In cell b, the one end point of the cell has been replaced with a pointed end instead of a flat surface. Additional cells where built with both ends cone shaped.

To test the effects of these changes, a vortex is induced on the wire, and then dislodged. If a reduction in cell diameter makes it easier for the vortex to come off the wire, than the vortex will favor coming off the thin end over the fat end. We can thus make qualitative judgments about the vortex behavior. In a similar manner, we can determine if the vortex prefers to dislodge off of a flat or pointed end.

When the vortex is dislodged and starts to precess around the cell, it's speed of precession is related to the cell's diameter. Thus, we can determine which end the vortex was dislodged, as a vortex dislodged from the the thinner end will precess faster than a vortex dislodged from the fatter end.

Currently, theory predicts that the vortices will favor coming off the end with the smaller diameter[4], because the distance a vortex must travel to move from the wire to the cell wall is smaller. In a similar manner, it is expected that the vortex will have an easier time dislodging from the cone shaped ends over the flat ends, as the transition from the wire to the wall is more gradual.

V. RESULTS

The majority of the summer was spent constructing the cells similar to the ones shown in figure 9. Unfortunately, a working cell was not successfully built until the tenth week of the program, so there was insufficient time to collect and analysis data. There are a number of reasons so much time was required to build a useable cell.

- Cells take a long time to build. Each cell has at least four gluing steps, and each gluing step requires a day for the glue to set and dry. Thus, a single cell has a minimum build time of four days.
- The cell wire is very fragile. Being less than ten micrometers in diameter, the wire is trivially easy to break, and it often does. It is not uncommon to invest three of four days in to a cell only to have the wire break during a gluing step.
- Testing is very time intensive. All cells are tested for leaks at both room temperature and at liquid helium temperature. While the room temperature test can be done relatively fast, it takes a day to cool down a cell, fill it with helium, evaluate its soundness, and return it to room temperature. Thus, only one cell can be tested per day.
- Cells are difficult to make. Superfluid helium has no viscosity, so it can climb though small holes or pores that would be impassable to a normal fluid. This makes producing a cell that is leak tight at superfluid temperatures difficult. Additionally, the interior wire sometimes electrically shorts or develops kinks. In either case, the cell must be discarded.

VI. COMPUTER HARDWARE AND SOFTWARE WORK

In addition to the work done building cells, the computer used in the data collection process was replaced. Previously, data was collected using LabVIEW on an old Windows computer, which was upgraded to a modern Linux machine. To adequately equip the new computer, several hardware interface cards were installed and configured.

In the transition from one operating system to another, some of the LabVIEW programs had to be rewritten. The windows VI's that were responsible for reading voltages failed to work on the Linux computer, and thus were replaced[6]. Additionally, the data fitting VI's had been deprecated and were also replaced.

REFERENCES

[1] C. R. Nave, *Hyperphysics*, Website (2006), URL http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html.
[2] *Encyclopædia britannica online*, URL www.britannica.com.
[3] L. Donev (2001), undergraduate Research Thesis.
[4] D. R. Zieve, Personal Communications (2007).
[5] The astute reader may realize that there is a singularity in the velocity at $r = 0$, which is clearly a physical impossibility. In a real helium superfluid, the vortex core is actually hollow and has a diameter of about an angstrom.
[6] The details of the transition from the Windows to Linux computer can be found in the document ”Hardware Install Journal”, available from Dr. Rena Zieve.