Mass Determination in Events with Multiple Missing Particles

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ABSTRACT

Decays that include multiple particles that interact only weakly and gravitationally arise in the Standard Model of particle physics, but are especially relevant to Supersymmetry models and to dark matter theories. Collider events that lead to such decays cannot be fully reconstructed from detector data, and they call for a new reconstruction method. This paper describes a novel method for the discrete determination of masses of these missing particles and all intermediate particles in the decay chain via the combination of multiple events into exactly-determined mathematical systems. It details the solutions for two such systems: (1) squark decay arising in R parity-conserving SUSY models at the LHC, and (2) standard model upsilon decay at SLAC.

This paper summarizes the research project in which I was involved as part of the Research Experiences for Undergraduates (REU) program at the University of California, Davis. It therefore includes an account of my personal contributions to this project.

1. Introduction

Many particle processes predicted both by the Standard Model (SM) of particle physics and by theories beyond the SM involve the production of weakly-interacting neutral particles, which cannot be detected in present or forthcoming particle colliders, such as the Tevatron at Fermilab, IL, the Stanford Linear Accelerator (SLAC) in Palo Alto, CA, and the Large Hadron Collider (LHC) at CERN, Switzerland. Reconstruction of the momenta of these "invisible" particles must therefore rely on a combination of data on the momenta of visible particles from the same decay chain and on mathematical techniques. Recently, some progress has been made towards finding new methods to address this issue. One approach [1,2] has been to examine mass distribution edges, possibly in combination with other methods that induce greater precision. An alternative idea [3,4] is to determine the missing masses by solving systems consisting of combinations of multiple events. The latter approach may require fewer events to reach the same, or better, precision, than the former. This paper analyzes a particular class of decay topologies – those that have "mirrored" chains of decay per event – using the multiple-event solution method. More specifically, for some topologies under a certain set of conditions this approach generates discrete mass solutions without necessitating endpoint analysis by forming and solving exactly-determined systems.

The search for successful reconstruction methods for events with two or more missing particles is largely motivated by the expectation that new physics processes of these types will take place at the energy range that will be available at the LHC when it becomes operational in 2008. In particular, various supersymmetry (SUSY) models predict the production of neutral final-state particles that only interact weakly and gravitationally in colliders. On the basis of theoretical considerations, it is possible that these particles may be produced at energies as low

[1] The research discussed in this paper was conducted as part of the Research Experiences for Undergraduates (REU) Program at the University of California, Davis under the guidance of and jointly with Hsin-Chia Cheng, Zhenyu Han, and Bob McElrath. My work was supported by the National Science Foundation.

as 100 GeV – 1 TeV, a range that will be accessible at the LHC.

While the SM has been remarkably successful in predicting experimental results, it leads to stabilization problems in the mechanism of electroweak symmetry breaking that is responsible for the generation of mass in elementary particles. The SM postulates a particle – the Higgs boson – that is responsible for giving mass to these particles; however, the theory fails to account for quantum divergences that arise in the mass term of the Higgs. In the absence of a satisfactory mechanism to account for and cancel these divergences, extreme fine-tuning of the bare mass term of the Higgs is necessary to reduce the mass to a level compatible with electroweak symmetry breaking. Such fine-tuning has no theoretical justification, and consequently various theories beyond the SM have been proposed to deal with the divergences – the so-called "hierarchy problem." Supersymmetry theories are largely motivated by the necessity to resolve the hierarchy problem, and they provide a mechanism for the cancellation of the Higgs divergences via the existence of new, heavy particles possessing a new type of symmetry[2]. In SUSY, every SM particle has a "super-partner" of different spin, but that is otherwise identical: SM bosons have fermionic superpartners with the suffix "-ino" attached (e.g. the gluon has the gluino as a superpartner), while SM fermions have bosonic superpartners with the prefix "s-" attached (e.g. squarks are the superpartners of SM quarks). Some super-particles, or "sparticles," are mixings of other sparticles. Neutralinos, for example, are mixings of wino and bino sparticles (both gauginos) and of the Higgsino; there are four such mixings, denoted as χ_i^0 where i is an index assigned in order of increasing mass. Neutralinos do not participate in the strong interactions, and as their name indicates, they are neutral particles so they do not interact electromagnetically. Therefore, if they are produced in a collider and if they are stable, they escape the collider undetected, leaving only a "missing momentum" signature.

SUSY models that conserve R-parity – a discrete multiplicative symmetry that takes on a R = +1 value for SM particles and R = -1 value for SUSY particles[3] – imply that the lightest supersymmetric particle (LSP) is absolutely stable. Since the LSP clearly cannot decay into other (heavier) sparticles, its only option is to decay into lighter SM particles; however, conservation of R-parity – which is multiplicative – forbids such decays, since $-1 \neq \prod(+1)$. In this case, if the LSP is also neutral, then it is an excellent candidate for the hypothesized Weakly-Interacting Massive Particle (WIMP) of cold dark matter[4]: it is neutral, stable on a cosmological timescale, and interacts only weakly and gravitationally. Some SUSY models postulate that the first neutralino, χ_1^0, is the LSP. Should this hold true, an accurate measurement of the mass of the χ_1^0 would not only shed light on SUSY models, but could also be used to gauge whether it is compatible with the estimated cold dark matter density in the universe, thus helping to confirm or reject theories in cosmology as well. An additional implication of R-parity conservation is that SUSY particles are always pair-produced in a collider: since the colliding particles are SM particles with even (+1) parity, they cannot produce an odd number of odd (-1) parity particles, but must produce an even number of such particles[5]. To summarize, then, in R parity-conserving SUSY models in which the χ_1^0 is the LSP, any SUSY decay chain must end in the χ_1^0 – invisible

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[2] Two other notable approaches to the hierarchy problem are Technicolor and large extra spatial dimensions.

[3] Defined as $R = (-1)^{2S+3B+L}$, where S is spin, B is the baryon number, and L the lepton number of the particle [5].

[4] Dark matter (cold and hot) is estimated to constitute about 22% of the universe and 85% of all matter in the universe. Dark energy is estimated to constitute about 74% of the universe and baryonic (regular) matter only 4% [6].

[5] The same logic applies to SUSY pair-annihilation.

to the detectors – and all SUSY events must contain an even number of these chains, compatible with the production of an even number of SUSY particles. Consequently, in these models SUSY events always contain at least two missing particles.

Supersymmetry, like electroweak symmetry, is a feature of high energies; it is broken at lower energies, leading to a mass discrepancy between SM particles and their superpartners when the superpartners become significantly more massive. If supersymmetry is to solve the hierarchy problem, then the supersymmetry-breaking scale cannot be much higher than the electroweak-breaking scale, i.e. at energies of 100 GeV – 1 TeV. In other words, if supersymmetric particle masses fall in this range – which will be accessible at the LHC in 2008 – then sparticles provide the necessary contributions to cancel the Higgs divergences. This theoretical consideration is the main motivator for the myriad of ongoing SUSY LHC phenomenology researches, including the work described in this paper.

This paper describes a novel method for finding discrete mass-space solutions for topologies that arise in R parity-conserving SUSY models where the LSP is the χ_1^0. Though the features of these topologies – two or more missing particles[6] together with several stages of decay and multiple visible resonances – are by no means limited to decays predicted by these models or exclusively to SUSY, their examination is motivated by these theories. While it is not possible to derive discrete solutions for missing particles' momenta in every decay topology, some topologies do have discrete solutions under the right mathematical manipulation. This paper discusses the types of topologies that can be solved discretely and describes the generic solution procedure (Section 2). It further describes the specific mathematical solutions of two such topologies: the first (Section 3) arising in SUSY models postulating R-parity conservation and a neutral LSP, and the second (Section 4) a SM meson decay that we hope to use to test our method.

This paper culminates my summer research work as part of the Physics Research Experiences for Undergraduates (REU) Program at the University of California, Davis, and I will therefore conclude with an account of my contributions to this project.

2. Topologies Permitting Discrete Mass-Space Solution

Let M represent the number of invisible – or missing – particles (those that escape the collider without detection) and I the number of intermediate particles (those that decay within the collider) in the topology (i.e. per event). If, based on the physics of the decay, we can make the assumption that the masses of corresponding intermediate particles are equal (i.e. in Figure 1, $M_x = M_{x'}$ and/or $M_y = M_{y'}$) then the calculations are considerably simplified and the number of events that must be correlated in order to find discrete solutions is reduced. Let S represent the number of such assumptions made, i.e. if we assume $M_x = M_{x'}$ but $M_y \neq M_{y'}$, then S = 1. In the class of toplogies discussed here, the invisible particles at the end of the decay chain (L and L') are all of the same type. This is not accounted for by this variable but is taken to be the default. Finally, let N represent the number of events necessary for finding a discrete solution for the topology. Note that in some cases the following derivation may either give a fractional solution for N or no solution for N, in which case the topology with the given assumptions cannot be solved discretely for any number of events. Mathematically, this is a consequence of the system

[6] This paper discusses only topologies in which the final-state missing particles are identical, because these are the topologies motivated by R parity-conserving SUSY models.

being either overdetermined or underdetermined for any number of events: an event combination of N events and M missing particles constitutes a system of 4MN variables (where the factor of 4 is due to the Lorentz four-momenta of each missing particle). To solve this system, it is necessary for the system to consist of 4MN linearly independent equations; this is not possible in every case.

2.1. The Quadratic Component

For the invisible particle *i*, we have $M_i^2 = p_i^2 = E_i^2 - \vec{p}_i^2$ in Lorentz notation[7]. Since final-state particles are stable, they are produced on their mass shell and have zero width in their mass distribution[8], and so all final-state particles of the same type have equal masses. In this paper, we discuss only topologies in which all final-state invisible particles are of the same type. If we have M such particles in each event, then we have $(p_{1,evti})^2 = (p_{2,evti})^2 = ... = (p_{\mathrm{M},evti})^2$, which gives M-1 quadratic equations in energy for each event. Moreover, the masses of the particles in all events are equal, e.g. for missing particle u, we have $(p_{\mathrm{u},evt1})^2 = (p_{\mathrm{u},evt2})^2 = ... = (p_{\mathrm{u},evtN})^2$, which gives N-1 equations for each missing particle, or M(N-1) for all missing particles. We have, therefore, a total of M(N-1)+M-1 quadratic equations in energy for N events and M missing particles per event.

2.2. The Linear Component

The linear component is composed of the missing momentum equations and the intermediate masses equations. The number of missing momentum equations depends on whether the decay takes place at a hadron or at a lepton collider. The equations governing the relations between the intermediate masses equations are collider-independent and are intrinsic to the topology itself.

The essential difference between hadron colliders, such as the LHC, and lepton colliders, such as SLAC, is the amount of information known about the initial state of the system. At a hadron collider, it is not possible to control the production energy of the desired initial-state particle because hadrons (protons, in the case of the LHC) are complex objects consisting of "partons" – valence quarks and a quark-gluon cloud. The exact fraction of total proton momentum occupied by each parton cannot be known and is only probabilistically described by parton distribution functions, which are derived experimentally. A hadronic collision is therefore a parton-parton collision with unknown momenta, and therefore the initial-state momenta along the beam line and the initial energy are unknown. Nonetheless, some initial-state momentum reconstruction is possible. Since the decaying particle is produced by partons moving along the beam line (z-axis), it has, to a good approximation, no transverse[9] momentum. Therefore, the final-state of the decay should similarly have zero transverse momentum. The solution, then, is to sum up all final-state momenta measurements along the x- and y- directions (individually); the transverse momenta of all missing particles is the negative of the sum, since the total transverse

[7] In Lorentz notation, $p_i = (E, p_x, p_y, p_z)$, and $p_i \cdot p_j = E_i E_j - \vec{p}_i \cdot \vec{p}_j$.

[8] All particles that decay have a certain width to their mass distribution; they may be produced at any mass within that distribution. Though normally the masses fall very closely to the distribution peak (the "classical" mass), there is some mass fluctuation. Particles may also be off their mass shell without violating energy conservation for a finite time due to the energy-time uncertainty relation. They are not "real" particles in that respect and must decay into other particles within the amount of time allowed by the uncertainty relation.

[9] Along the x- and y- axis of the collider; perpendicular to the direction of beam propagation.

momentum must be zero. Hence, at a hadron collider we have for each event: $p_{miss,x} = -\sum_{visible} p_x$ and $p_{miss,y} = -\sum_{visible} p_y$. This gives 2N equations for each event combination from the missing momentum at a hadron collider.

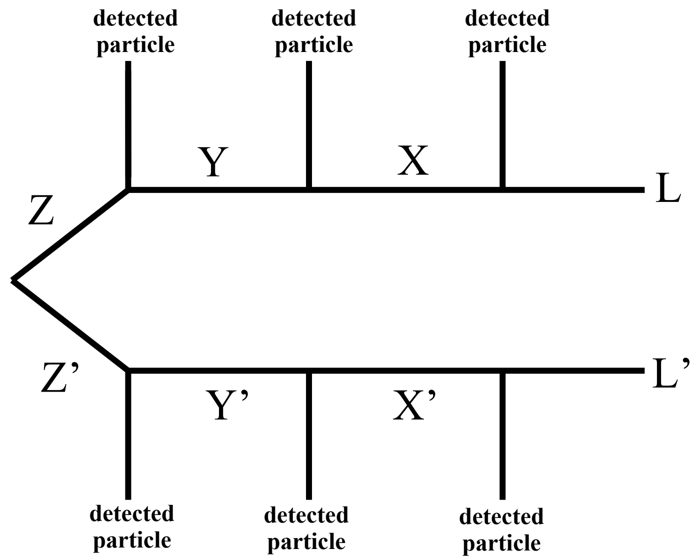

FIGURE 1: Sample topology of mirrored-chain decays and multiple missing particles

At a lepton collider the initial-state can be fully known. Since leptons are not composite objects, setting the initial beam energy determines entirely the collision energy. Particle *i* can be produced on-shell by setting the collision energy to its on-shell mass, in which case we know the full four-momenta of the decay and have the following equations for each event: $p_{miss,x} = p_{initial,x} - \sum_{visible} p_x$, $p_{miss,y} = p_{initial,y} - \sum_{visible} p_y$, $p_{miss,z} = p_{initial,z} - \sum_{visible} p_z$, and $E_{miss} = E_{initial} - \sum_{visible} E$. Hence we have 4N equations for each event combination from the missing momentum at a lepton collider.

The remaining linear equations come from the relations between the intermediate particle masses, i.e. the masses of particles X, X', Y, Y', Z, and Z' in Figure 1. Suppose that intermediate particle X decays into particles i = 1, 2, ..., n. Then $M_x{}^2 = (p_1 + p_2 + ... + p_n)^2 = (\sum_i p_i)^2$, where p_i is the Lorentz four-vector. Ignoring for the moment finite-width effects and off-shell resonances[10], the masses of intermediate particle X in all events are equal. So we have

[10] The probability of a particle being off-shell decreases as its mass moves farther from the on-shell (classical) mass. In some models, the width of the mass distributions of all relevant particles is very small, with the greatest width on the order of a few GeV (normally for squarks). In those cases it is possible to ignore these effects in the mathematical analysis and then use cuts in the final mass distributions (see Section 3.3) to account for these small effects. However, other models may have substantially more pronounced finite-width effects for some particles, in which case this analysis may be highly inaccurate or entirely invalid.

$\sum_i p_{i,evt1}{}^2 = \sum_i p_{i,evt2}{}^2 = ... = \sum_i p_{i,evtN}{}^2$, or N-1 equations for each intermediate particle type. If there are I intermediate particles, then we have I(N-1) such equations. If, due to the physics involved, we can make the assumption that any two corresponding intermediate particles in the same event are of the same type or of equal mass (i.e. $M_x = M_{x'}$ and/or $M_y = M_{y'}$) then we have one additional equation ($p_X{}^2 = p_{X'}{}^2$) for each such assumption. For S such assumptions, we have I(N-1)+S. Notably, since the missing particles all have equal masses, then when any of the intermediate particle equations is expanded and squared, the quadratic terms $p_1{}^2, p_2{}^2, ..., p_n{}^2 = M_1{}^2, M_2{}^2, ..., M_n{}^2$ cancel each other out, causing the equations to have only a linear dependence on $p_1, p_2, ..., p_n$.

The system will have a discrete solution if the number of equations equals the number of variables 4MN. Solving for N by setting the number of equations to the number of variables 4MN generates the following scenarios:

Hadron collider, equality of intermediate masses not assumed: N = (1+I)/(2-3M+I)
Hadron collider, all corresponding intermediate particles have equal masses: N = (1+I-S)/(2-3M+I) where S is half the number of intermediate particles (S is even by default).
Lepton collider, equality of intermediate masses not assumed: N = (1+I)/(4-3M+I)
Lepton collider, all corresponding intermediate particles have equal masses: N = (1+I-S)/(4-3M+I) where S is half the number of intermediate particles (S is even by default).

Therefore, for the scenario of two missing final-state particles and six intermediate particles at a hadron collider, where all intermediates are assumed to be equal (i.e. S = 3), we can solve for the missing momentum discretely if we form combinations of two events. I.e. each system consists of a pair of events. On the other hand, the scenario of two missing final-state particles and four intermediate particles has no discrete solutions at a hadron collider because 2-3M+I always equals zero. However, it has an integer-N solution at a lepton collider in certain cases; for example, if S = 1, then we have a solution for N = 2.

3. Topology I: Squark Pair-Production at the LHC

Assuming R-parity conservation, squarks are pair-produced at a proton-proton collision, leading to two decay chains in each event. In this topology (Figure 2), each squark subsequently decays into the second neutralino (χ_2^0) and a SM quark. The SM quark hadronizes and exits the collider as a jet, which is then detected by the detector subsystems and is reconstructed via jet reconstruction algorithms. The χ_2^0 further decays into a lepton and a slepton; the lepton exits the collider and is detected, and the slepton undergoes a further decay into a lepton (which also exists the collider and is detected) and the first (lightest) neutralino – the χ_1^0 – which is the LSP in the model and which undergoes no further decays due to R-parity conservation. What we have, therefore, are two mirrored chains of decay, with six visible particles (four leptons and two quarks), six intermediate particles that decay inside the collider (the two squarks, two χ_2^0's, and two sleptons, where we assume no finite-width or off-shell particles, i.e. equal masses of corresponding particles on each chain), and two final-state missing particles – the two χ_1^0's.

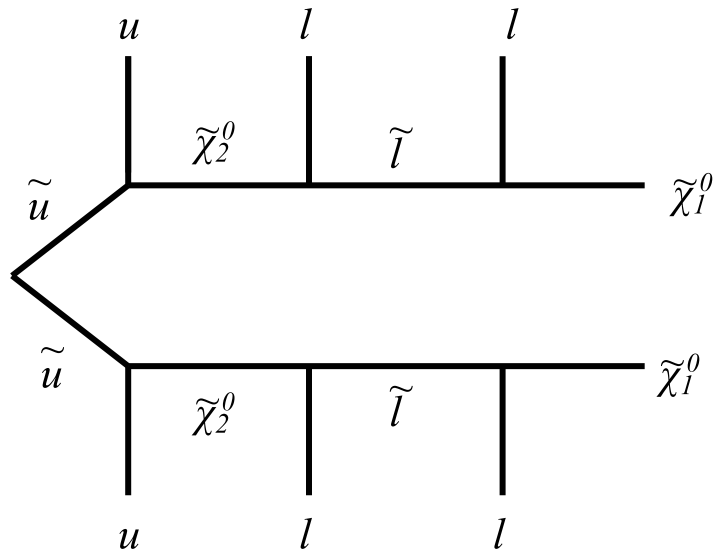

FIGURE 2: Squark pair-production followed by mirrored-chain decay

3.1. Mathematical Solution of the Topology

From the derivation in Section 2, at the LHC – a hadron collider – under the assumption of completely mirrored chains, i.e. all masses of corresponding intermediate particles are equal, we can solve the topology discretely with combinations of two events. We have sixteen variables (refer to Figure 3): p_1, p_2, p_a, p_b (where p = (E,px,py,pz)), and sixteen equations. The quadratic system is as follows:

$$\begin{array}{lcl} p_1^2 = p_2^2 & & E_1^2 - \vec{p}_1^{\,2} = E_2^2 - \vec{p}_2^{\,2} \\ p_1^2 = p_a^2 & \text{---->} & E_1^2 - \vec{p}_1^{\,2} = E_a^2 - \vec{p}_a^{\,2} \\ p_1^2 = p_b^2 & & E_1^2 - \vec{p}_1^{\,2} = E_b^2 - \vec{p}_b^{\,2} \end{array}$$

Since $M_{13}^2 = M_{24}^2 = M_{ac}^2 = M_{bd}^2$ by assumption,

$$\begin{array}{lcl} (p_1 + p_3)^2 = (p_2 + p_4)^2 & & 2(p_1 \cdot p_3 - p_2 \cdot p_4) = p_4^2 - p_3^2 \\ (p_1 + p_3)^2 = (p_a + p_c)^2 & \text{---->} & 2(p_1 \cdot p_3 - p_a \cdot p_c) = p_c^2 - p_3^2 \\ (p_1 + p_3)^2 = (p_b + p_d)^2 & & 2(p_1 \cdot p_3 - p_b \cdot p_d) = p_d^2 - p_3^2 \end{array}$$

because $p_1^2 = p_2^2 = p_a^2 = p_b^2$ by assumption, and so these terms cancel out. The above three equations can be written as:

$$2(E_1 \cdot E_3 - \vec{p}_1 \cdot \vec{p}_3 - E_2 \cdot E_4 + \vec{p}_2 \cdot \vec{p}_4) = M_4^{\ 2} - M_3^{\ 2}$$
$$2(E_1 \cdot E_3 - \vec{p}_1 \cdot \vec{p}_3 - E_a \cdot E_c + \vec{p}_a \cdot \vec{p}_c) = M_c^{\ 2} - M_3^{\ 2}$$
$$2(E_1 \cdot E_3 - \vec{p}_1 \cdot \vec{p}_3 - E_b \cdot E_d + \vec{p}_b \cdot \vec{p}_d) = M_d^{\ 2} - M_3^{\ 2}$$

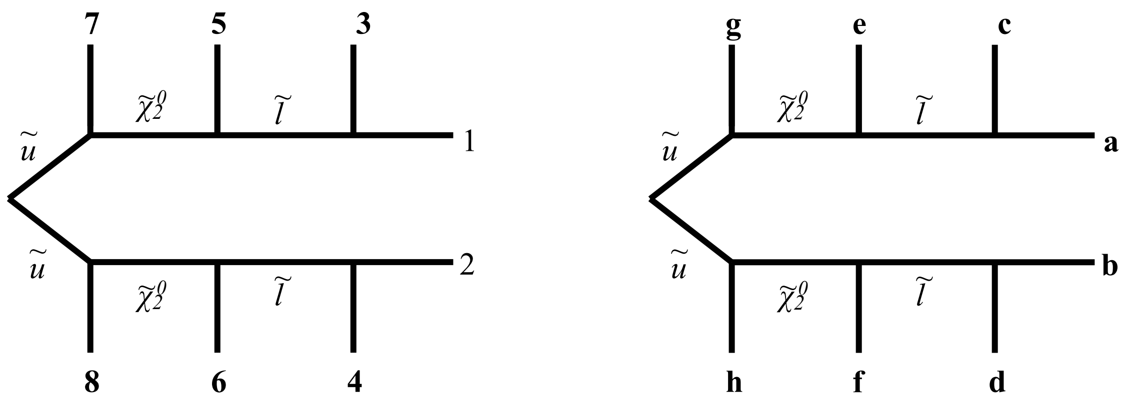

FIGURE 3: Labeling for the two arbitrary events in each combination

Similarly, since $M_{135}^{\ 2} = M_{246}^{\ 2} = M_{ace}^{\ 2} = M_{bdf}^{\ 2}$ by assumption, we have:

$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_2 \cdot (E_4 + E_6) + \vec{p}_2 \cdot (\vec{p}_4 + \vec{p}_6)] = M_4^{\ 2} + M_6^{\ 2} - M_3^{\ 2} - M_5^{\ 2}$$
$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_a \cdot (E_c + E_e) + \vec{p}_a \cdot (\vec{p}_c + \vec{p}_e)] = M_c^{\ 2} + M_e^{\ 2} - M_3^{\ 2} - M_5^{\ 2}$$
$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_b \cdot (E_d + E_f) + \vec{p}_b \cdot (\vec{p}_d + \vec{p}_f)] = M_d^{\ 2} + M_f^{\ 2} - M_3^{\ 2} - M_5^{\ 2}$$

and from $M_{1357}^{\ 2} = M_{2468}^{\ 2} = M_{aceg}^{\ 2} = M_{bdfh}^{\ 2}$ we have:

$$2[E_1 \cdot (E_3 + E_5 + E_7) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5 + \vec{p}_7) - E_2 \cdot (E_4 + E_6 + E_8) + \vec{p}_2 \cdot (\vec{p}_4 + \vec{p}_6 + \vec{p}_8)] =$$
$$= M_4^{\ 2} + M_6^{\ 2} + M_8^{\ 2} - M_3^{\ 2} - M_5^{\ 2} - M_7^{\ 2}$$
$$2[E_1 \cdot (E_3 + E_5 + E_7) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5 + \vec{p}_7) - E_a \cdot (E_c + E_e + E_g) + \vec{p}_a \cdot (\vec{p}_c + \vec{p}_e + \vec{p}_g)] =$$
$$= M_c^{\ 2} + M_e^{\ 2} + M_g^{\ 2} - M_3^{\ 2} - M_5^{\ 2} - M_7^{\ 2}$$
$$2[E_1 \cdot (E_3 + E_5 + E_7) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5 + \vec{p}_7) - E_b \cdot (E_d + E_f + E_h) + \vec{p}_b \cdot (\vec{p}_d + \vec{p}_f + \vec{p}_h)] =$$
$$= M_d^{\ 2} + M_f^{\ 2} + M_h^{\ 2} - M_3^{\ 2} - M_5^{\ 2} - M_7^{\ 2}$$

Finally, we have the four missing tranverse momentum equations:

$$p_{1,x} + p_{2,x} = p_{miss,xevtI} = -(p_{3,x} + p_{4,x} + p_{5,x} + p_{6,x} + p_{7,x} + p_{8,x})$$
$$p_{1,y} + p_{2,y} = p_{miss,yevtI} = -(p_{3,y} + p_{4,y} + p_{5,y} + p_{6,y} + p_{7,y} + p_{8,y})$$
$$p_{a,x} + p_{b,x} = p_{miss,xevtII} = -(p_{c,x} + p_{d,x} + p_{e,x} + p_{f,x} + p_{g,x} + p_{h,x})$$
$$p_{a,y} + p_{b,y} = p_{miss,yevtII} = -(p_{c,y} + p_{d,y} + p_{e,y} + p_{f,y} + p_{g,y} + p_{h,y})$$

First we solve the linear system in terms of three of the energies (the choice of which three of the four energies is arbitrary). This allows us to rewrite the three quadratic equations in a total of three variables. Given the size of the linear system, an analytic solution is cumbersome and not easily found. Numerically, the 13 by 13 system is easily solved by a computer program employing simple linear algebraic routines (refer to Section 3.2). An analytic solution to the quadratic system does not exist[11]; in fact, even good numerical solutions are not easy to program, and they exhibit various problems (refer to Section 3.2). Once the values for the three energies have been found, these can be substituted into the solution for the linear system to solve for the remaining momenta components of the missing final-state particles, and hence also for the intermediate particles, in each pair of events.

Solving for the missing four-momenta becomes significantly more involved when the assumption that the detected visible momenta are perfectly matched to the corresponding particles in the topology is removed. In reality, it is not possible to infer from the data which lepton momenta correspond to a particular diagram lepton, and it is only known which leptons are positively- and negatively-charged and what type they are (e.g. electrons or muons). To allow for all possibilities in the data, it is necessary to form combinations of the detected momenta for each event. Due to charge conservation, in each of the mirrored decay chains the two leptons must have opposite charges. Therefore, if lepton 5 is positive, lepton 3 must be negative, which leaves a dual freedom for lepton 6; however once the charge of lepton 6 is picked, the charge of lepton 4 must be the opposite. If all leptons are of the same type, then the result is sixteen possible combinatorics for each event, or $16\times16=256$ combinatorics per pair[12]. If a total of N events are analyzed, we have N(N-1)/2 pairs, or a total of 128N(N-1) combinations. For the large number of events (N ≥ 1000) generally desired for reasonable statistics, the solution process may become very time-consuming, and highly effective programming algorithms are essential for reducing the solution time involved while retaining the accuracy of the solutions.

Another idealization that was assumed in the mathematical solution is the equality of the masses of all intermediate corresponding particles. I.e. we assumed that all squarks have the same mass, that all the χ_2^0's have the same mass, and that all the sleptons have the same mass. However, due to finite-width effects and off-shell resonances this is not exactly true. Note that since the χ_1^0's are stable, they both have zero-width mass distributions and are on-shell, and therefore the assumption that they have equal masses is exactly correct.

Additionally, the smearing of momenta caused by detector resolution effects further contributes to wrong statistics and widens the final mass distributions by adding off-peak solutions.

3.2. Computational Work

The code that solves this topology is composed of several files written in C++ with ROOT[13] classes that perform the tasks of event generation (simulation), input and output management, linear system derivation and solution, construction of the quadratic system, quadratic system solving and the discarding of negative or complex solutions, and substitution of the quadratic solutions into the remaining momenta equations for full event reconstruction.

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[11] This was established by Abel's Impossibility Theorem, which states that a polynomial equation of fifth degree or higher does not have an algebraic solution [7].

[12] If we are to analyze processes in which the leptons are of mixed types, then the number of needed combinations is reduced due to fewer degrees of freedom.

[13] Particle analysis software developed at CERN, Switzerland [8].

For the ideal scenario, the main code uses the TPythia ROOT class to generate events. This is an interface class in C++ to the Fortran-77 Monte-Carlo event generation program Pythia [9]. An alternate code uses ATLFast[14] to generate events in a more realistic collider environment simulation. Data storage in either of these programs is undertaken via the use of C++ structures that define a single event, an event combination (i.e. a pair of events), and a solution set. These structures are outputted to binary files, which can be read by executing a separate program.

The main code pairs up all the events and forms all 128N(N-1) combinations, extracts the visible four-momenta from the relevant structures, and constructs the two matrices – of dimensions 13 by 13 and 13 by 4 respectively – used to solve the linear system for the $\vec{v} = (\vec{p}_1, \vec{p}_2, \vec{p}_a, p_b)$ vector of unknowns. The 13 by 13 matrix is inverted and multiplied by the 13 by 4 matrix to generate the solutions for $\vec{v}$ in terms of E_1, E_2, and E_a. At this point the remaining equations are the three quadratics, which now contain only constants and the variables E_1, E_2, and E_a. The quadratic solver accepts the quadratic equations in matrix format; if Q is a matrix for the i^{th} quadratic, then $(E_1, E_2, E_a, 1) \cdot Q \cdot (E_1, E_2, E_a . 1)$ equals the i^{th} quadratic equation. The code uses the linear solution to construct these matrices and send them to the solver.

We currently have two types of solvers that can interact with the main program to generate solutions for the quadratic equations. One is based on homotopy methods, and the second uses algebraic reduction and elimination methods to reduce the three quadratics to a single univariate ninth-order polynomial, which is then solved numerically by the polynomial root solver Rpoly_ak1 [11] (based on the Jenkins-Traub method). Both these solvers employ Newton's method to refine the roots that they find. At present, these solvers are still undergoing further improvement. They have both been shown to miss a small percentage of the solutions (≈0.1%) due to machine precision problems. Although this is below the error due to detector resolution and other experimental effects, we hope to further reduce it.

The solvers eliminate any solution sets (where a "set" consists of a (E_1, E_2, E_a) constant vector) containing complex energy solutions before sending them to the main code. The main code then accepts all the real solution sets[15], some of which may contain negative energy solutions as well as solutions that are positive in E_1, E_2, and E_a but negative in E_b. The main code processes all solution sets to extract the ones that contain only positive energies, and then uses those to solve for the remaining 4-momenta ($\vec{v}$, above). All this information is stored in the structures discussed above.

3.3. Data Analysis and Results

Once the quadratic solvers are deemed satisfactory, the solution code will be run on the order of 1000 simulated events in the ideal scenario of no smearing and no finite-width effects or off-shell resonances as well as in scenarios including all of these effects. Due to the large number of combinatorics, this will be a computing power-intensive task that will necessitate the use of multiple computers over the course of several days (with the exact amount of time dependent on which solver has been chosen).

[14] ATLFast is a ROOT-based fast Monte-Carlo simulation program for the ATLAS detector at the LHC [10].

[15] There are zero such sets in the case that all solutions are complex. The solver using homotopy methods may return up to eight real solutions (in the case that none of the solutions are complex); the second solver may return up to nine real solutions (in the case that none of the solutions are complex) because although a system of three quadratics should have eight solutions, due to the reduction of the system to a ninth-order polynomial there are nine solutions.

As a quick initial test of the code, some mass distribution plots were generated both in the ideal scenario and with momentum smearing and finite-width effects using the second version of the main code (refer to the previous section). A few of these are shown in Figures 4-7. The solutions plotted in these figures were generated by the second quadratic solver discussed above (that reduces the equations to a ninth-order univariate polynomial). The figures clearly show that adding smearing, finite-width, and off-shell effects widens the distributions. Moreover, though running the code with all combinatorics generates more wrong solutions (thus increasing the width of the distributions), the peak at the correct mass point is clearly present even when the combinatorics are added, as long as there are no smearing, finite-width effects, or off-shell particles. Once these effects are added (through the ATLFast software), there are no longer sharply-defined peaks, and it becomes necessary to fit the distribution to a curve. We have not yet done any fitting, but once more events are generated that will be done as well. It is also possible to narrow the distributions by running the mass solutions through certain cuts. One possibility for such a cut would be to place an upper limit on the difference between the four particles' masses based on information from mass-difference plots.

Finally, note that though Figures 6 and 7 both display data with smearing, finite-width and off-shell effects, and all combinatorics, the distributions in Figure 7 are narrower because there are fewer combinatorics in the case of leptons of different types.

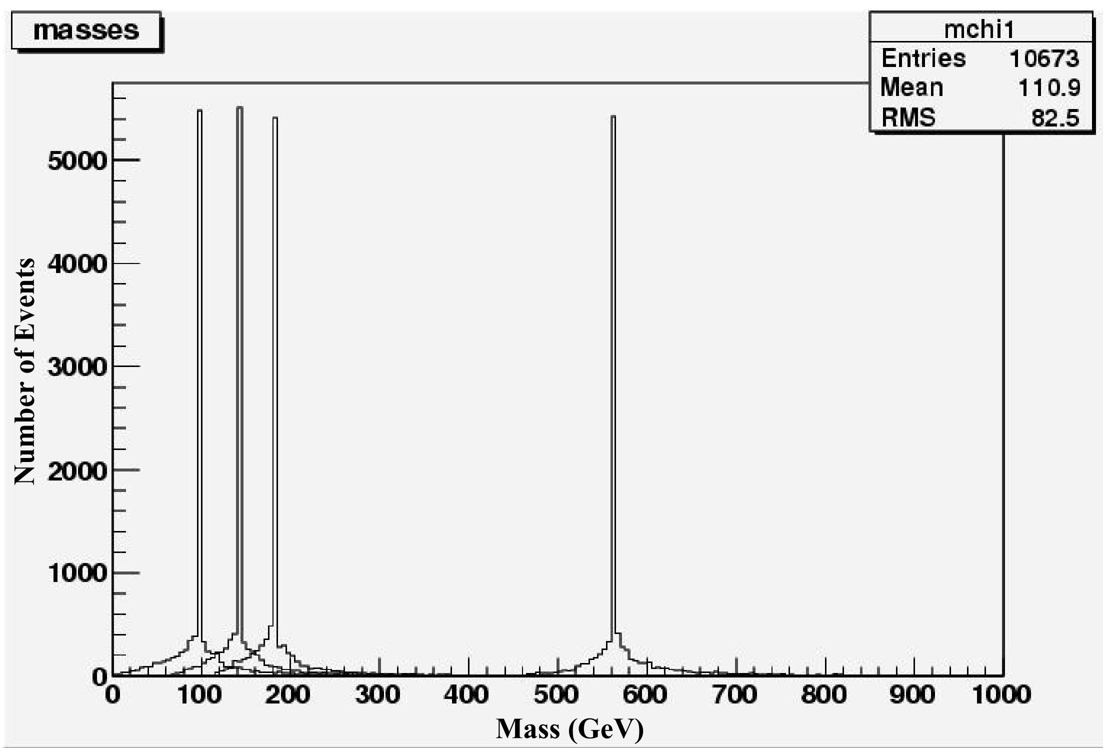

FIGURE 4: Mass distributions for the (right to left) squark, slepton, χ_2^0, χ_1^0 for the correct combination only and with no momentum smearing and no finite-width effects. All particles are on-shell, and all four leptons are muons.

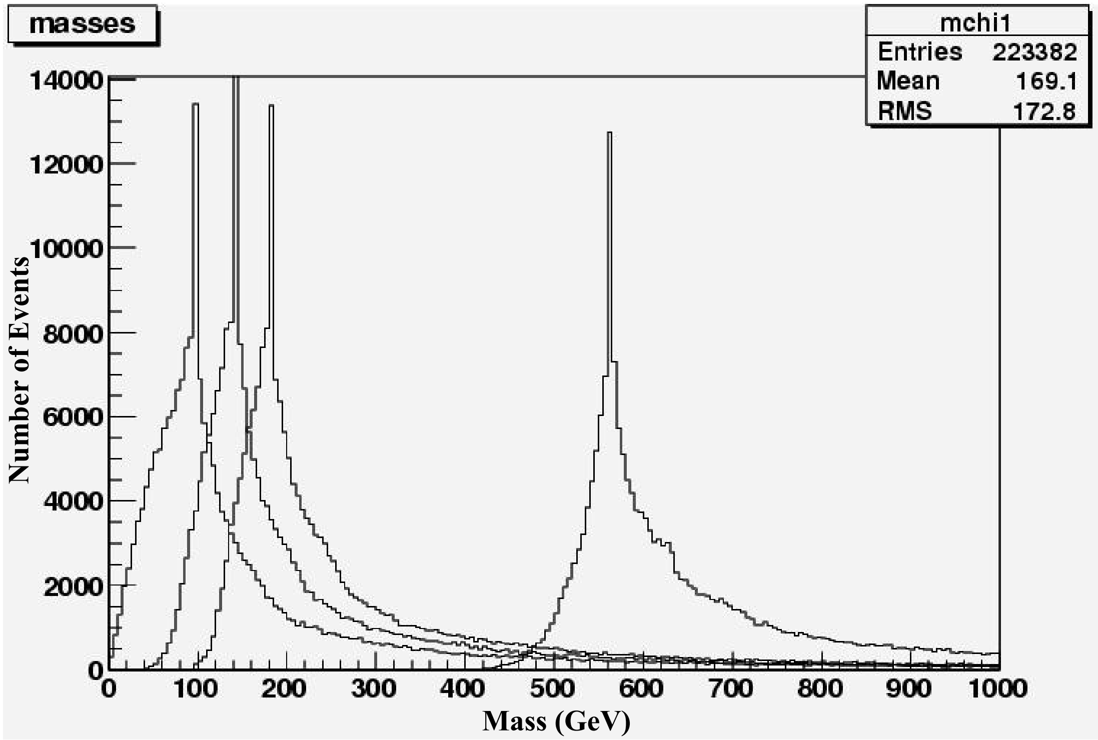

FIGURE 5: Mass distributions for the (right to left) squark, slepton, χ_2^0, χ_1^0 for all 256 combinatorics/pair, but with no momentum smearing and no finite-width effects. All particles are on-shell, and all four leptons are muons.

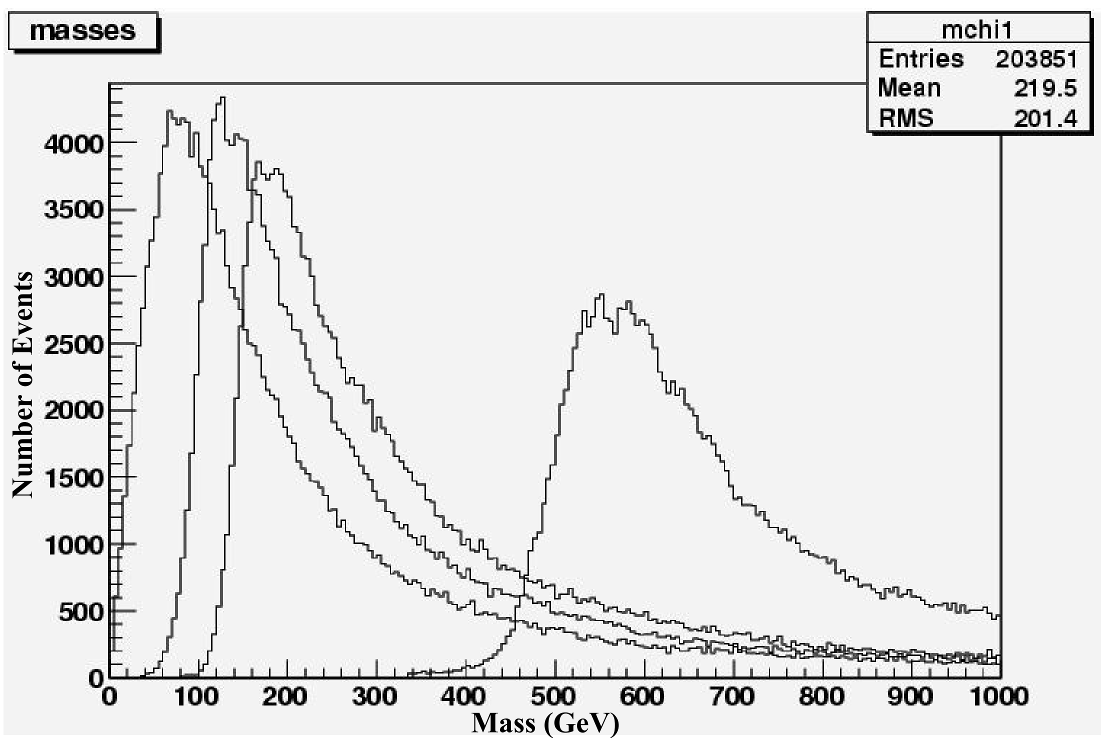

FIGURE 6: Mass distributions for the (right to left) squark, slepton, χ_2^0, χ_1^0 for all 256 combinatorics/pair with momentum smearing and with finite-width effects. Particles may be off-shell. All four leptons are muons.

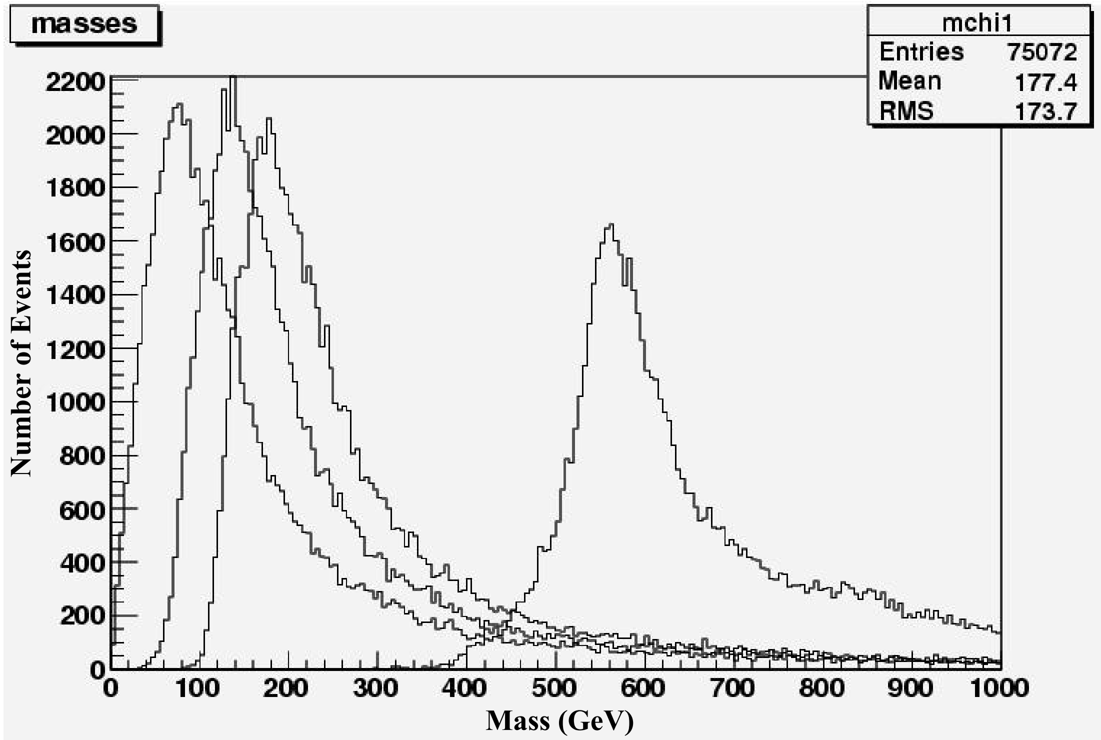

FIGURE 7: Mass distributions for the (right to left) squark, slepton, χ_2^0, χ_1^0 for 16 combinatorics/pair (two of the leptons are muons and two are electrons) with momentum smearing and with finite-width effects. Particles may be off-shell.

4. Topology II: Upsilon Decay at SLAC

In this decay, a singly-produced upsilon decays into two mirrored chains of $B_0 \rightarrow D_0X \rightarrow B_0 l\upsilon$, where X is some visible (SM) particle. Since the masses of the B and D mesons and the neutrino are experimentally known, the goal in solving this topology is to test our method of deriving solutions via combinations of multiple events. In this case there are two invisible final-state particles (the neutrinos), four intermediate particles (the B and D mesons) and four visible particles – two leptons and two other (X and X') identical particles (refer to Figure 8). We expect to accumulate the data for this decay at SLAC, a lepton collider, and therefore the initial-state momenta of the upsilon is fully known. With these parameters, the derivation in Section 2 shows that this topology can be solved if we assume that one pair of corresponding masses is equal (e.g. the masses of the two D mesons are equal) and combine events into pairs, which results in a system of 16 equations in 16 variables (the linear system contains 13 equations in 13 variables, and the quadratic system contains 3 equations in 3 variables), just as in the topology in Section 3. Note that, unlike the squark decay topology in Section 3, the upsilon decay topology has no discrete solution at a hadron collider for combinations of any number of events, because the denominator 2-3M+I is always zero (M = 2, I = 4).

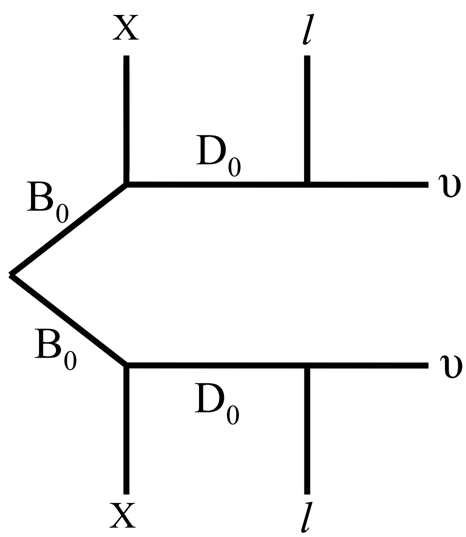

FIGURE 8: Upsilon decay via two mirrored chains of B_0→ D_0X → B_0*lυ*, X visible particle

Due to its similarity with the topology in Section 3, this topology will not be described in as much detail. First, the quadratic system here is identical to that in Section 3. If we assume that the B mesons have equal masses but make no such assumptions about the D mesons, then we have three linear equations from the masses of the B mesons ($M_{135}^{2} = M_{246}^{2} = M_{ace}^{2} = M_{bdf}^{2}$):

$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_2 \cdot (E_4 + E_6) + \vec{p}_2 \cdot (\vec{p}_4 + \vec{p}_6)] = M_4^{\,2} + M_6^{\,2} - M_3^{\,2} - M_5^{\,2}$$
$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_a \cdot (E_c + E_e) + \vec{p}_a \cdot (\vec{p}_c + \vec{p}_e)] = M_c^{\,2} + M_e^{\,2} - M_3^{\,2} - M_5^{\,2}$$
$$2[E_1 \cdot (E_3 + E_5) - \vec{p}_1 \cdot (\vec{p}_3 + \vec{p}_5) - E_b \cdot (E_d + E_f) + \vec{p}_b \cdot (\vec{p}_d + \vec{p}_f)] = M_d^{\,2} + M_f^{\,2} - M_3^{\,2} - M_5^{\,2}$$

And two such equations from the masses of the D mesons ($M_{13}^{\,2} = M_{ac}^{\,2}$ and $M_{24}^{\,2} = M_{bd}^{\,2}$):

$$2(E_1 \cdot E_3 - \vec{p}_1 \cdot \vec{p}_3 - E_a \cdot E_c + \vec{p}_a \cdot \vec{p}_c) = M_c^{\,2} - M_3^{\,2}$$
$$2(E_2 \cdot E_4 - \vec{p}_2 \cdot \vec{p}_4 - E_b \cdot E_d + \vec{p}_b \cdot \vec{p}_d) = M_d^{\,2} - M_4^{\,2}$$

Finally, we have eight missing momenta conditions:

$$p_{1,x} + p_{2,x} = p_{initial,x} - p_{final,xevtI} = p_{initial,x} - (p_{3,x} + p_{4,x} + p_{5,x} + p_{6,x})$$
$$p_{1,y} + p_{2,y} = p_{initial,y} - p_{final,yevtI} = p_{initial,y} - (p_{3,y} + p_{4,y} + p_{5,y} + p_{6,y})$$
$$p_{1,z} + p_{2,z} = p_{initial,z} - p_{final,zevtI} = p_{initial,z} - (p_{3,z} + p_{4,z} + p_{5,z} + p_{6,z})$$
$$E_1 + E_2 = E_{initial} - E_{final,evtI} = E_{initial} - (E_3 + E_4 + E_5 + E_6)$$

$$p_{a,x} + p_{b,x} = p_{initial,x} - p_{final,xevtII} = p_{initial,x} - (p_{c,x} + p_{d,x} + p_{e,x} + p_{f,x})$$
$$p_{a,y} + p_{b,y} = p_{initial,y} - p_{final,yevtII} = p_{initial,y} - (p_{c,y} + p_{d,y} + p_{e,y} + p_{f,y})$$

$$p_{a,z} + p_{b,z} = p_{initial,z} - p_{final,zevtII} = p_{initial,z} - (p_{c,z} + p_{d,z} + p_{e,z} + p_{f,z})$$
$$E_a + E_b = E_{initial} - E_{final,evtII} = E_{initial} - (E_c + E_d + E_e + E_f)$$

With very few modifications, the same program used to solve the topology in Section 3 can be used to generate solutions here. For the upsilon decay it will be necessary to use a simulation software other than Pythia. We will be using the EvtGen software[16] for event simulation, and we hope to generate collider data from SLAC for testing the code and the general method on a known system. The beams will be collided at an energy of 10.58 GeV, which is the mass shell of the upsilon (4S). The upsilon decays into $B_0\,\bar{B}_0$ or B^+B^- 100% of the time[17]. Since the neutrino, D meson, and B meson masses are known, the accuracy of our solutions for these masses will provide an estimate for the error involved in solving for other topologies (in which the invisible particles' masses are truly unknown).

5. Personal Contributions and Conclusion

Most of my time on this project has been devoted to deriving the linear solutions and constructing the quadratic matrices for the two topologies that are the focus of this paper (with greater emphasis on the first) and to writing the version of the main code that generates events using the TPythia class, forms all 128N(N-1) combinations of momenta, solves the linear systems, constructs the quadratic matrices, sends these matrices to either of the external quadratic solvers, discards all negative-energy solutions that are returned by the solver, solves for the four-momenta with the energy solutions, and stores all event, combination, and solutions information in binary files (which can be read by another program that I wrote). The number-of-events derivation in Section 2 is also my work. The plots shown in Section 3.3 of this paper were generated with the version of the main code written by Zhenyu Han, which uses the linear solution and quadratic matrix construction portions of the version of the code that I wrote[18].

The work that is described in this paper will continue to undergo development both in terms of algorithmic and computational efficiency and in the larger sense of generalization to other topologies. The energy-scale that will be accessible at the LHC offers hope for the discovery of new physics phenomena and may provide evidence for supersymmetric models and dark matter, among other theories. In order to make as complete as possible use of the anticipated data from the LHC, it is essential to be able to fully analyze events with topologies like those described here. In the case of the SUSY decay discussed in Section 2, an accurate determination of the mass of the LSP would enable researchers to discover whether it is the long-sought cold dark matter particle, the WIMP. Achieving accurate solutions of such topologies may therefore provide the answers to some of the most profound questions in particle physics and cosmology.

[16] EvtGen is an event generator designed for the simulation of the physics of B decays at SLAC [12].

[17] The B^+B^- decay is very similar to the $B_0\,\bar{B}_0$ decay, and so this analysis applies in the same way to that decay.

[18] The versions are currently adapted to each interface with a different solver; the version that I wrote interfaces with the homotopy methods solver.

Acknowledgments

My work this summer could not have been performed without the support of my advisor, Hsin-Chia Cheng, and of Zhenyu Han and Bob McElrath. I am indebted to them for their assistance and steadfast guidance as I proceeded through this project.

This work was supported by the National Science Foundation as part of the Research Experiences for Undergraduates (REU) Program at the University of California, Davis.

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