How the Cookie Crumbles

Data Analysis for Experimental Granular Materials Research

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Abstract:

Granular materials are characterized as a conglomeration of macroscopic particles which experience a loss of energy upon interaction, while being large enough to avoid thermal motion fluctuations. Despite centuries of study, their behavior is still mysterious. Theoretical work has traditionally focused on spheres and hard ellipsoids rather than simple polygons or polyhedra. Previous experimental research by Dr. Zieve on polygons created from lattices of welded ball bearings had indicated a link between angle and density in two dimensions immediately prior to an avalanche. The goal of this project was to confirm those findings and attempt to determine what portion of the material was responsible for the correlation. Code was written to analyze photos of a rotating frame containing a sample granular material, making it possible to find density and angle for regions of arbitrary size and calculate correlations from them. Correlations for various granular shapes were most visible when taken over large regions and decreased in smaller regions as noise became dominant with one notable exception.

1. Introduction

The Wikipedia defines a granular material as “A conglomeration of discrete solid macroscopic particles characterized by a loss of energy whenever the particles interact. The constituents that compose a granular material must be large enough such that they are to not subject to thermal motion fluctuations.” Classic examples of granular materials include sand, grain, and sugar. Granular materials are found throughout everyday life. Though the modern study of granular materials is hundreds of year old, very little about their behavior is understood, despite efforts by Faraday, Coulomb, and many others. Granular materials do not easily fit into the standard classifications of matter as solids, liquids, or gases. Though made up of solids, granular materials can flow like liquids, prompting some to suggest that they are in fact a state of matter in their own right. Granular materials form the basis of many industries, having applications in pharmaceuticals (pills and powders), in agriculture (grain, seed, fruits), and construction (gravel, concrete, etc.). [1]

Granular materials exhibit bizarre, counter-intuitive properties. In a tall cylinder (such as a grain silo), the pressure on the base does not increase infinitely as height increases; rather it reaches a maximum value and then the walls of the container must bear any additional weight. Cubes have a maximum packing density of 1, as they can be placed directly adjacent one another with entire faces in contact. However, on deposition, they have a packing density of only 0.68. Irregularly shaped particles are less dense than spheres on deposition, yet compress well under vibration; by contrast, spheres resist compression during vibration. [1]

Theoretical work in this area has concentrated principally on packing hard ellipsoids or mixtures of spheres , rarely has it dealt with other shapes such as simple polygons or polyhedra. [2]

In order to cut the problem down to a scale that is appropriate for study, a number of simplifications must be made. Two dimensional behavior could provide insight toward three dimensional problems, yet is simpler to work with. In two dimensions, uniform spheres pack into triangular lattices, the densest possible arrangement. Computer simulations with shapes such as pentagons and heptagons have shown that they form doubles lattices, making them impractical for the study of random close-packed behavior. [2]

For the purpose of this study, all shapes were created out of 1/8” ball bearings, welded into variations of triangular lattices (Figure 1).

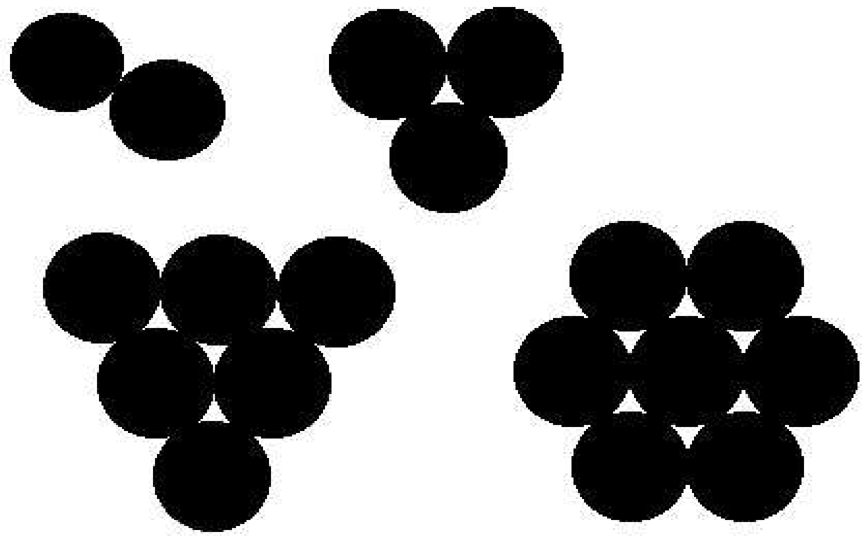

Figure 1

The ball bearings, hereafter referred to as balls, were placed in a circular frame containing an irregular border. The balls were confined to a single layer with a piece of Plexiglases, and the apparatus was rotated, causing avalanches of balls (Figures 2 and 3).

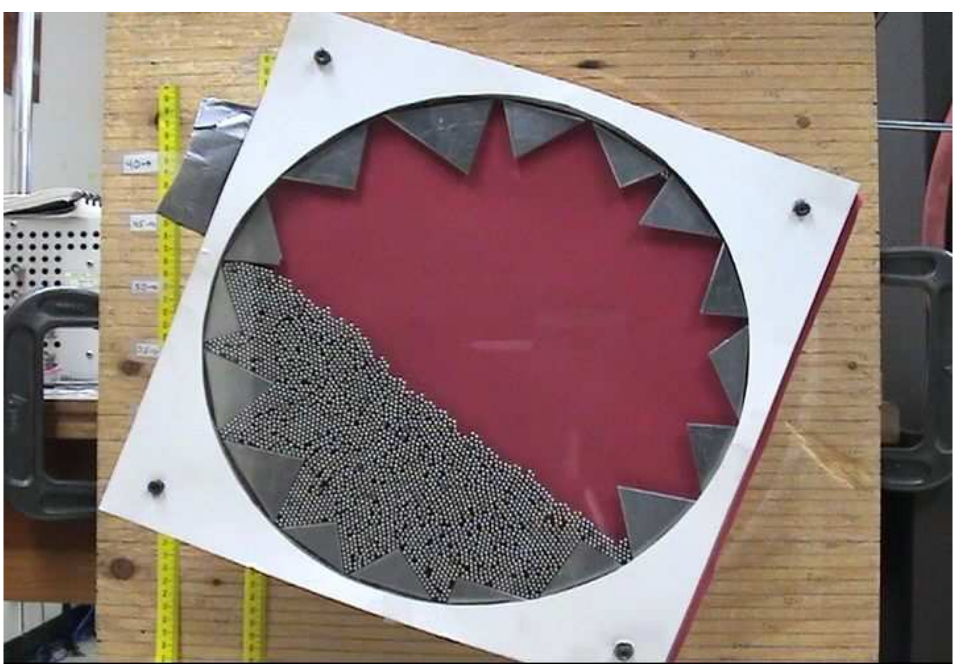

Figure 2

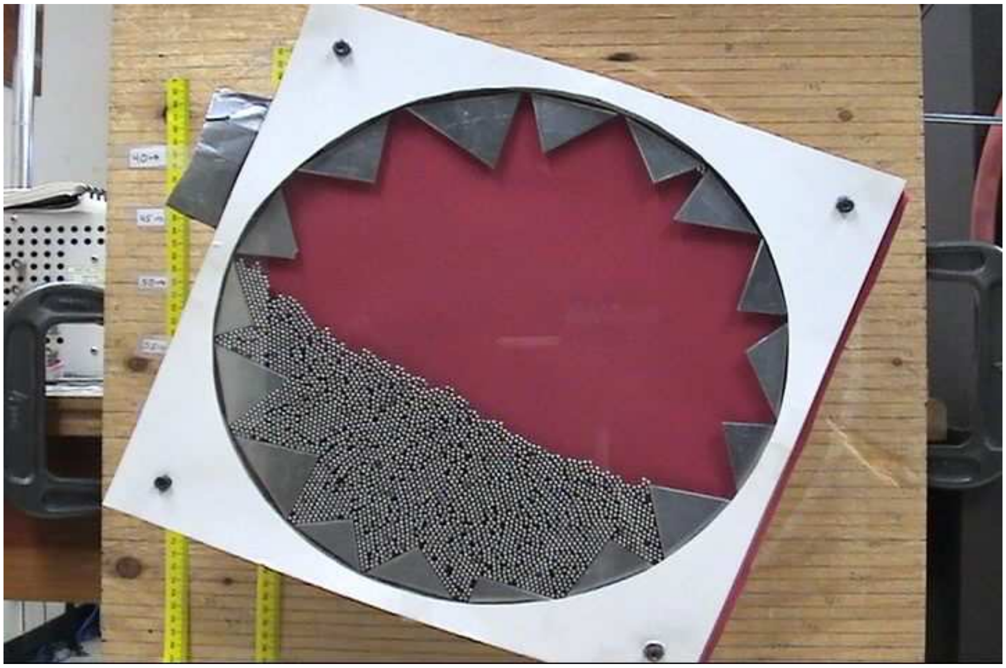

Figure 3

In a previous study, photos were taken of the apparatus directly before and after successive avalanches for a variety of shapes, including small and large triangles, large diamonds, doubles, and hexagons. IDL code had been written to calculate the density and angle immediately prior to the avalanche for the entire area containing balls. The total mass of balls was divided by the known mass of a single ball, giving the number of balls contained in the apparatus. The area was found through radial integration. The density was then found by dividing the number of balls by the area. Plots of this data revealed a correlation between density and angle. This method had a number of shortcomings, foremost of which was that it could only be applied to the entire area of balls, and not smaller regions. Because of that, it could not be used to determine what portion of the balls might be responsible for the correlation. Given that between consecutive avalanches, the vast majority of the balls do not move (being below the level of avalanche), it seemed that the correlation was likely caused by a thin strip of balls along the slope; additional code was required to verify this.

2. Methodology

Study of the digitized pictures seemed to reveal that balls showed up as a 3x3 matrix, a bright pixel surrounded by somewhat dimmer pixels. Based on this observation, code was written to locate bright pixels and count the number of balls component pixels located inside an arbitrary area. That total number was then divided by nine to yield the number of total balls contained in the specified region. The area of the specified region was found by counting the number of pixels contained within the borders of the region. Various versions of this code can calculate density and angle for small central regions using both regular and irregular borders, top and bottom halves of the ball area, as well as the entire frame. The code was tested on small and large triangles, big diamonds, hexagons, and doubles. Figure 4 shows the entire frame outlined in purple, the top and bottom "halves", defined by the green line, and the strip near the top edge believed to be the principal source of the correlation, outlined in light blue.

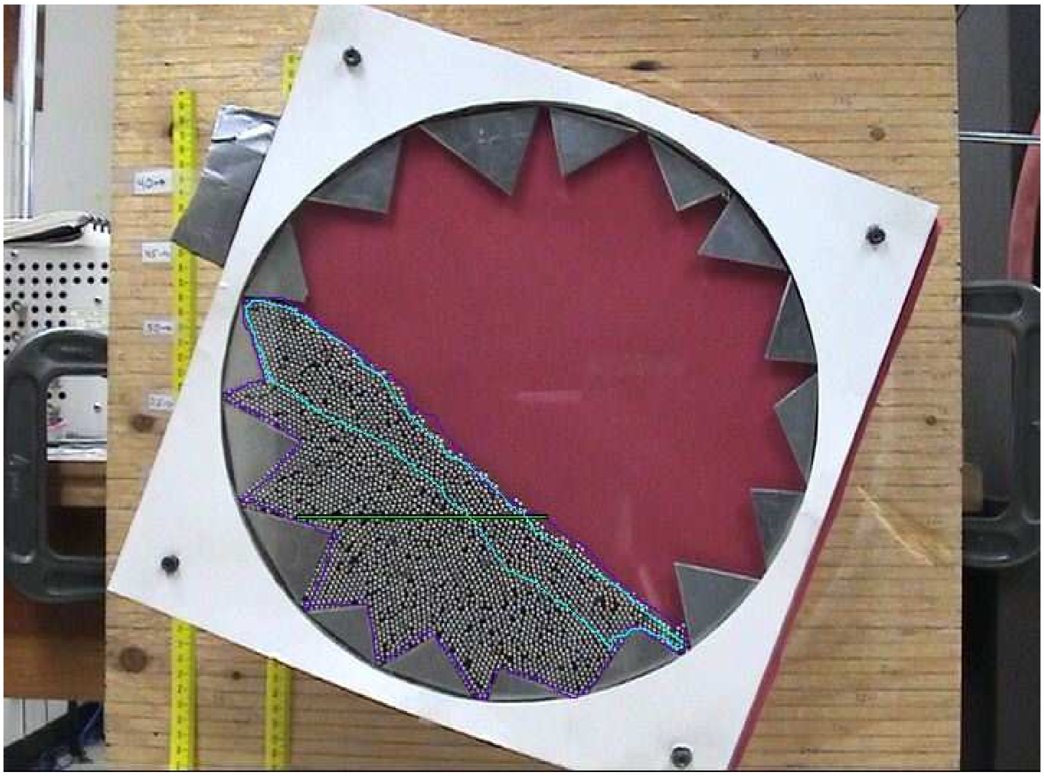

Figure 4

3. Data & Results

Trials with the earlier code had found that the correlation was highest with small triangles, approximately 0.5. Therefore, it seemed reasonable to test the accuracy of the new code against the old. The correlation found using the new code for an entire frame of small triangles was 0.46579, very close to the value obtained with the older method (Figure 5).

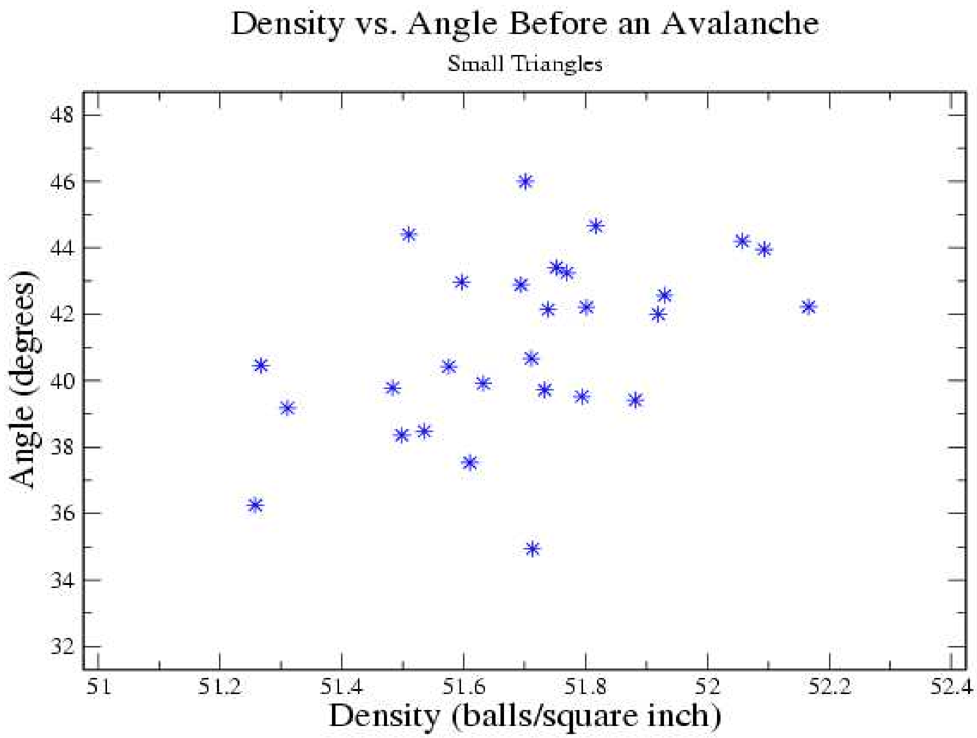

Figure 5

This appears to confirm the results of previous research. However, the goal of this project was to find densities and angle for small regions. Therefore, code was written to divide the photo horizontally, approximately halfway between the highest and lowest ball locations. Densities were obtained for the regions above and below the line (Figures 6 and 7).

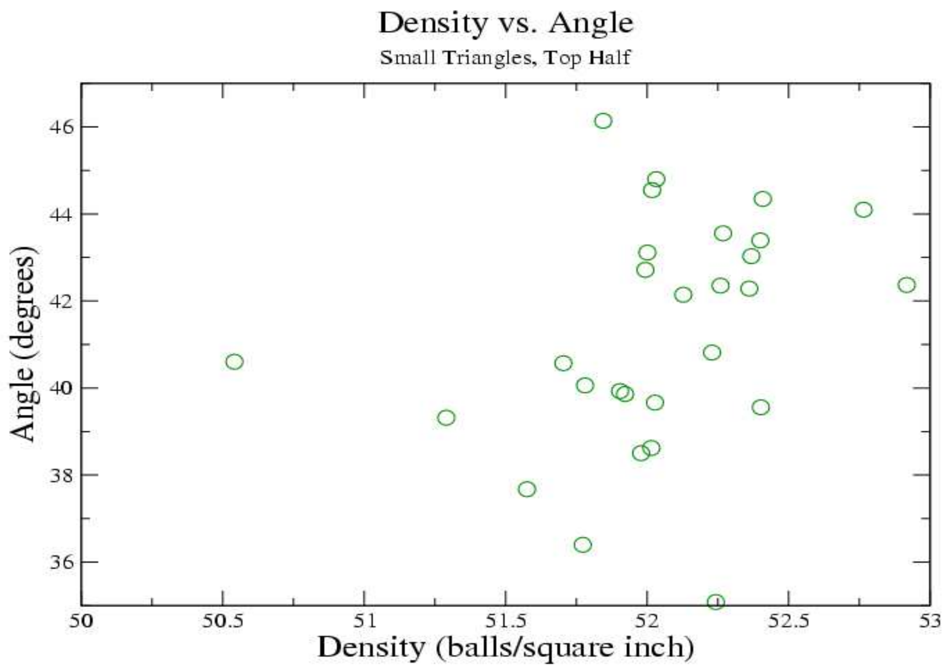

Figure 6

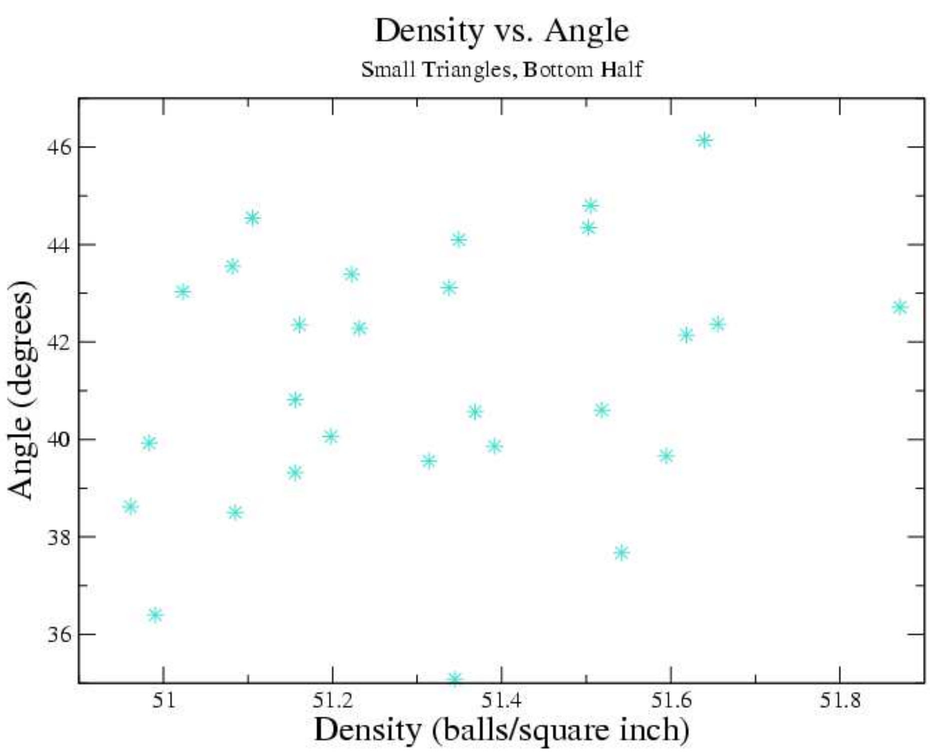

Figure 7

As expected, the top and bottom “halves” each have a smaller correlation than the whole frame, at 0.312038 and 0.267438, respectively. Given the location of the horizontal division, it would be expected that the top “half” would have a slightly higher correlation than the bottom “half”, as it contains somewhat more of the area believed to be responsible for the correlation. The data appears to confirm this.

Arbitrary small regions, however, proved problematic. They showed tiny positive correlations, at best, and negative correlations, at worst. This would seem to indicate that noise becomes dominant for small regions. There was an exception to this unforeseen setback, however. The big diamonds each contained one data point that, when removed, revealed a moderate but noticeable correlation.

Region	Correlation
5 to 15	0.273065
5 to 35	0.312940
5 to 50	0.346340
15 to 35	0.196939
15 to 50	0.293854
35 to 50	0.284407

Table 1

Table 1 shows that the strongest correlation was found for the region 5 to 50 pixels from the upper edge of the ball region, with a correlation of 0.346340. The correlation increases as the region depth increases, encompassing larger portions of the strip suspected to be principally responsible for the correlation. The correlation generally decreases the upper edge of the regions descends. It remains unclear why only the big diamonds reveal this correlation for arbitrary small regions, and this merits further study.

4. Analysis & Conclusions

Small regions exhibit negligible correlation of angle and density with the notable exception of the big diamond shapes. The top and bottom halves of the small triangles also have noticeable correlation, though less than for the whole frame. The whole frame has the strongest correlation which is similar the value calculated in earlier research. Given that the correlation is most visible when taken over larger areas and decreases over smaller areas, it seems likely that noise is dominating the smaller regions. This is consistent with the big diamond trials for small regions, in which a small correlation was found. Additional testing is required to confirm these suppositions.

5. Future Work

Minor modifications of the code would be required to find densities and angles for the entire strip believed to be causing the correlations. The basic framework is already written, but there was insufficient time to debug it before the end of the summer. Analysis of this region could prove whether this is indeed the critical area for correlating density and angle.

Works Cited

1. H.M. Jaeger, S.R. Nagel, and R.P. Behringer, “Granular Solids, Liquids, and Gases”, *Reviews of Modern Physics,* October 1996

2. I.C. Rankenburg and R.J. Zieve, "The Influence of Shape on Ordering of Granular Systems in Two Dimensions", *Physical Review E*, October 2001