Effects of Gaussian Curvature on Vortex Pinning in Superfluid Helium-4

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Abstract

This project explores the effect of Gaussian curvature upon the pinning of vortices within superfluid Helium-4. This was tested in a small cylindrical chamber of He-4, which is cooled down to ~0.5 K. It is then rotated several turns to create vortices within the chamber, and then held still. A wire runs the length of the chamber, and it is energetically favorable for a vortex to be on the wire. The amount of vortex on the wire is detected by bathing the wire in a magnetic field and pulsing the wire with current, and then analyzing the envelope of the emf created by the changing flux. The wire should be repelled from the top of the bump because of its positive Gaussian curvature, and should attach at some lower point on the slope of the bump, where the repulsion and increased energy-per-length of a free vortex should even out. We should see fractured semi-stable levels in-between N=1 and N=0, where N=1 is a full vortex on the wire, and N=0 is no vortex on the wire. We could also see some precession about the same radial level on the bump.

Introduction

First discovered in 1908, superfluid helium has been clearly elucidated into a well-known phenomenon. At approximately 2.17 Kelvin helium-4 goes “superfluid” (for He-3, the lambda temperature is around 0.001 Kelvin at ambient temperature[1]). One of the most remarkable effects of going superfluid is that the viscosity of the helium goes to zero.

Naturally, this is a very interesting non-Newtonian effect which is applicable to a number of experiments and studies. One of these experiments is the observation of vortices within the superfluid helium-4.

A vortex is created around fluid flow, and is very hard to model physically and mathematically. In fact, for all but the simplest vortices, the Navier-Stokes equation is almost impossible to solve[1]. In light of this, the next step in understanding vortices is to make the experiment as simple as possible, so as to get a very basic understanding first with hopes to extend it to larger and more complicated phenomena.

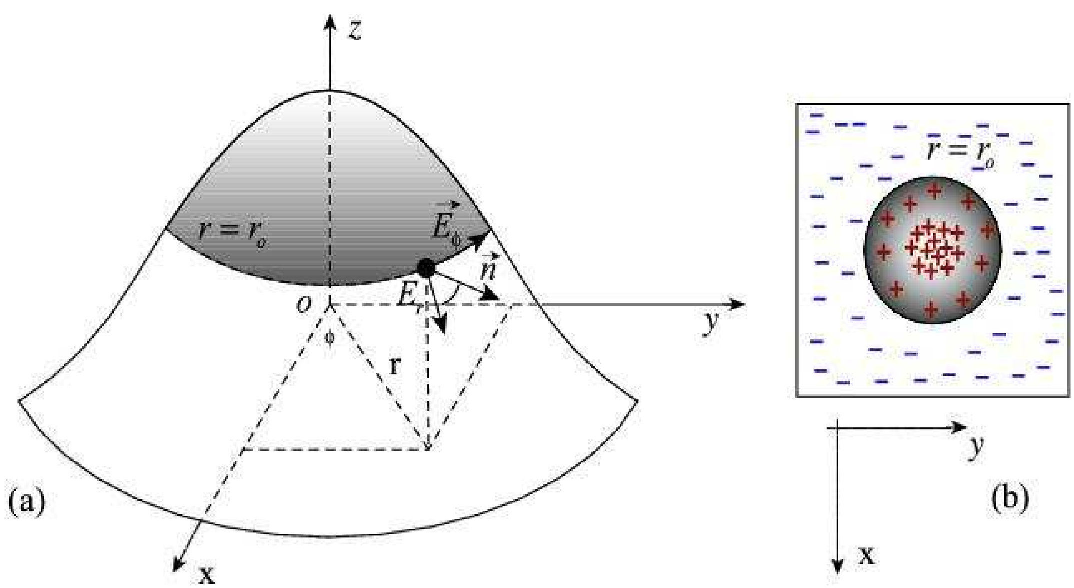
A common occurrence when dealing with vortices in superfluid He-4 is that a vortex will become pinned to a spot in its container. It may de-pin quickly, or it may stay stuck for many, many hours. There were so many pinnings that it begged further study.

Our project was based off a paper by Vincenzo Vitelli and Ari Turner[2] made a mathematical prediction that vortices will have an attraction/repulsion to the surface of their substrate based on whether that surface has positive or negative Gaussian curvature. Giving more theoretical framework was an additional paper by Vitelli and David Nelson[3].

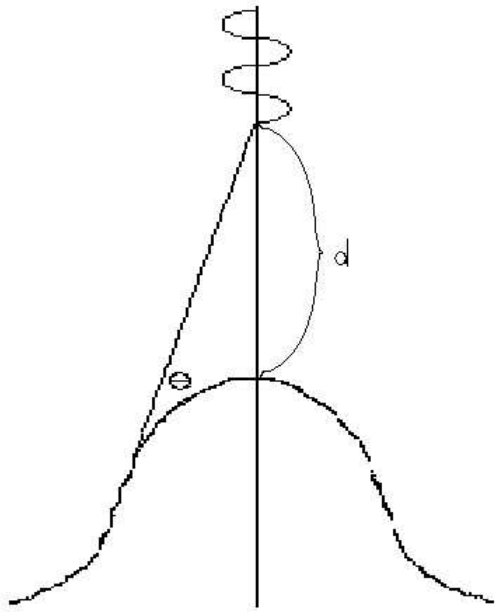
Background

First we should explore this idea of Gaussian curvature. The Gaussian curvature at any one given point is going to be product of the two principal curvatures, k_1 and k_2,at that self-same point. The principal curvatures are the maximum and minimum bending values at this given point, and so the Gaussian curvature determines whether a surface is locally convex (when it is positive) or locally saddle (when it is negative). Also, the Gaussian curvature will be larger at points with more curvature then elsewhere. Gaussian curvature is given, using radial coordinates. So at the top of the bump, the Gaussian curvature will be positive. However, at some point further down the bump, one of the principal curvatures will switch signs (on a hemisphere, it would be at the very bottom of the bump), and the Gaussian curvature will be negative.

The paper by Vitelli et al posits that there will be an attraction/repulsion from positive/negative Gaussian curvature[2]. In the figure below [2], you can see how the "charge" from the curvature is distributed. There is some r_0 where the Gaussian curvature changes signs, and at that point the repulsion becomes an attraction.



To model this system, we had two incorporate two competing energy costs: geometric potential and energy-per-length for the vortex. It takes significantly less energy per length, a factor of 3, for the vortex to be on the wire rather than be free in the cell chamber. This is due to the fact that if a vortex is on the wire, its core is supplanted by the wire itself. So the vortex will not want to be off the wire, but the geometric potential can be so strong, especially at the top of the bump, that it will still push the vortex further down the slope of the bump. In the end, it should find some middle point on the bump where the geometric potential and increased length of the vortex even out. So now the vortex doesn't cover the whole wire, and will attach at some point further up on the wire.



This decreased coverage should be detectable by our methods [see methodology]. The larger the d is, the easier it will be to detect.

Methodology

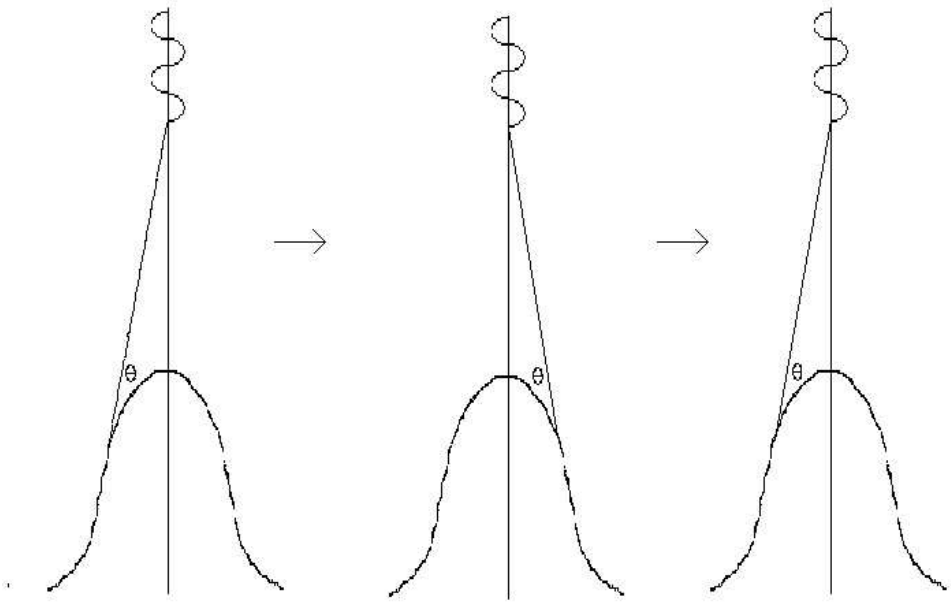
The actual experiment is designed around a thin cylindrical cell. It has a height of 50.9 mm and a diameter of 7 mm, and is made of copper. A single strand of Niobium-Titanium superconducting wire runs the length of the cell, connected at the center of both cylinder tops. One end is glued shut with a Sty-cast cap, and the other end is only partially blocked so as to allow superfluid He-4 to flow into the cell. To simulate our Gaussian bump, we put a small Stycast blob at the fully capped end. The Stycast bump was roughly hemispherical in shape and had a diameter of around 3 mm, though it had a deviation of around 1 mm. To get the best results, we had bumps that had an aspect ration of in-between ~0.7 and ~1. The wire ends up running through the bump, so its centered orientation is undisturbed. This cell is on the end of a cryostat, a large fridge, that we lower into a dewar full of helium. To get a vortex on the wire, we rotate the cryostat within the dewar. A matrix of vortices forms as we spin the cryostat, and once we stop rotating it, they hopefully coalesce into a vortex that is centered on the wire.

To detect a vortex on the wire, we first use superconducting magnets to set up two magnetic fields in the direction of the normal modes of vibration of the wire. Then the wire is once pulsed with current. This causes the wire to vibrate, and the fluid pressure of the vortex on the wire will tug it around. By examining the envelope of this emf which was created by the changing flux in the magnetic field, we can see how much of the wire is covered by a vortex.

But how do we actually cool the helium to our very low target temperature of ~0.5 Kelvin? The helium comes at around ~4.2K, so we then use the cryostat to pump the helium to 1.5 K in the 1 K pot. When we say pump, we mean reduce the vapor pressure, which subsequently reduces the temperature. We then put the He-4 in thermal contact with the He-3, which is a much rarer and expensive isotope. It also has much lighter atomic weight, so we can pump it down to much lower temperature, and get the He-4 down to ~0.5 K[1].

Conclusion/Further Tests/Anticipated Results

At least one viable cell with the Stycast bump has been created, so tests can and will proceed in the near future. What do we hope to see? If all goes well, we can hope to see two possible signals. One would be fractured semi-stable levels of signal between N=1 and N=0. N=1 would indicate that the wire was completely covered by a vortex, where as N=0 would mean that there was no vortex at all on the wire. If a vortex finds a point, or even multiple points, on the bump where it can pin, we would see the signal pause at some in-between value. The vortex, due to fluid flow in the cell, will precess around, slowly unraveling off the wire, making the signal go down from N=1 to N=0[1]. During the precession, the vortex may find several of these points on the bump – giving us several of these semi-stable levels. Another possible signal, even more distinctive than the other (and shown below), would be if the precession of the vortex was around the same radial level on the bump, giving us a stable signal at the same N level. That would be a powerful indication of vortex pinning, therefore a powerful vindication of the model and the theory.



[Figure] – The sine wave oscillation is due to the wire's circular vibration and the magnets' alignments.

Works Cited

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