Problem Set 3
Physics 204B

Due in class Monday January 27, 2014
Late HW accepted until class January 29

1. Evaluate
   a) \( \int_{0}^{\infty} \frac{\cos \pi x}{1+x^2+x^4} \, dx \)
   b) \( \int_{0}^{2\pi} \frac{\sin^2 \theta d\theta}{5+3\cos \theta} \)

2. Show that:
   a) \( \int_{0}^{\pi/2} \frac{d\theta}{a+\sin^2 \theta} = \frac{\pi}{2\sqrt{a^2+a}} \) (a > 0)
   b) \( \int_{0}^{\infty} \frac{\sin^2 x}{x^2} \, dx = \frac{\pi}{2} \)
   c) \( \int_{-\infty}^{\infty} \frac{\sin^3 x}{x^3} \, dx = \frac{3\pi}{4} \)
   d) \( \int_{-\infty}^{\infty} \frac{\cos bx - \cos ax}{x^2} \, dx = \pi(a-b) \) for a, b > 0
   e) \( \int_{-\infty}^{\infty} \frac{x^2 \, dx}{(x^2+2x+2)^2} = \pi \)
   f) \( \int_{0}^{\infty} \frac{\sin \pi x}{x^3-x} \, dx = \frac{\pi}{4}(e^{-\pi} - 3) \)
   g) \( \int_{-\infty}^{\infty} \frac{\cos ax \, dx}{(x^2+b^2)^2} = \frac{\pi(1+ab)e^{-ab}}{2b^3} \) (a, b > 0)

3. a) Evaluate \( \int_{-\infty}^{\infty} \frac{dx}{\cosh x} \), using a rectangular contour with one side along the real axis and another at Im \( z = i\pi \).
   b) Could you do part a) using a rectangle with one side on the real axis and another at Im \( z = -i\pi \)? Could you do it with a large semicircle closing in the upper half-plane? Explain your answers.
   c) Evaluate \( \int_{-\infty}^{\infty} \frac{x^2 \, dx}{\cosh x} \). This is tricky, but your answer to part a) will help.

Extra problem not to turn in: This question has you relate two Taylor expansions of the same function in the region where they both converge. You should do part a). The calculation in part b) is a bit ugly, so I leave it to you to decide how much of it to do.
   a) Expand \( \frac{1}{z+i} \) about \( z = 0 \) (to cubic terms in \( z \)) and about \( z = 2 \) (to cubic terms in \( z-2 \)).
      What is the radius of convergence in each case? One of the expansions converges at the center point of the other; which one?
   b) Although the two series you found may appear different, they must coincide at points inside the radius of convergence of both, since they converge to the same function. Verify that the constant terms are indeed the same: in the \( z = 2 \) expansion, multiply out the powers of \( z-2 \) to find ALL constant terms. The constant terms form an infinite series, and the pattern should become apparent after the first few terms. Sum this infinite series; the answer should equal the constant term in the \( z = 0 \) expansion. You could carry out similar sums for the \( z, z^2, \) and \( z^3 \) terms. Why does this type of calculation only show that the series coincide within the radius of convergence?